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Improved Acquisition for System Sustainment: Availability-Based Importance Framework for Maintenance, Repair, and Overhaul Acquisition

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Kash Barker, Ph.D.

University of Oklahoma

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Final Report
Improved Acquisition for System Sustainment: Availability-Based Importance Framework for Maintenance, Repair, and Overhaul Acquisition

Prime Offeror Institution
University of Oklahoma

Technical Contact
Kash Barker, Ph.D.

Administrative/Business Contact
Lawana Dillard

RESEARCH SUMMARY

Availability of aging DoD weapons systems is of significant concern, particularly as budgets tighten and system replacement is infeasible. This work developed availability-based importance measures to (i) focus on critical components and (ii) identify appropriate suppliers for acquisition related to system sustainment.

Task 1. Availability-Based Importance Measures. An effective defense strategy requires aircraft, among other weapons systems, to be available and ready for use when circumstances deem necessary. The same is true for a competitive strategy in a manufacturing environment (i.e., available equipment for production), among other environments. This task offers a set of importance measures to identify the critical components in a system from their influence on system achieved availability, a common availability calculation that is a ratio of mean time between maintenance and total system time, including mean maintenance time. With these measures, more effective maintenance plans, including inspection and supply inventory, can focus on those components that more significantly impact achieved availability. The ability to prioritize components based on their importance to system availability will ultimately facilitate the MRO acquisition process through effective maintenance triggers and supplier selection

Task 2. Stochastic Availability Importance Measures. The availability importance measure developed in Task 1 makes use of point estimates representing mean time between maintenance and mean maintenance time. However, expected values are merely point estimates that ignore valuable information about the entirety of the distribution. This task explored the treatment of the point estimate parameters of availability with probability distributions and proposed a novel multicrtieria decision analysis technique to compare the distributions to determine a ranking of component importance.

Task 3. Improving Inspections and Comparing Suppliers. The availability of aging systems, particularly weapons systems within the Department of Defense, is of significant concern as budgets tighten and system replacement is infeasible. This work addresses the selection of sole suppliers according to their ability to provide component parts that strengthen availability of the system. We extend a popular multi-criteria decision making approach, TOPSIS, by (i)

considering the availability of individual components as the criteria in the decision problem and (ii) weighting those criteria according to the value of component importance measures while (iii) accounting for uncertainty in underlying reliability and maintainability parameters with interval numbers. An aircraft example illustrates the approach.

Task 4. Illustrative Examples. Illustrative examples for all three tasks have been inspired by conversations with employees at Tinker Air Force Base, though due to the sensitive nature of such systems, no real data could be used.

RESEARCH OUTPUT

The remainder of this report provides the primary methodological developments and research findings of the funded work, provided in the form of scholarly journal manuscripts. Complete references across all tasks are provided at the end.

In total, the following papers and presentations were submitted or are still in progress (with support acknowledged).

1. Gravette, M.A. and K. Barker. 2014. Achieved Availability Importance Measures for Enhancing Reliability Centered Maintenance Decisions. *Journal of Risk and Reliability*, **229**(1): 62-72.
2. Hague, R.K., K. Barker, and J.E. Ramirez-Marquez. 2015. Interval-valued Availability Framework for Supplier Selection Based on Component Importance. Accepted in *International Journal of Production Research*.
3. Shaffer, R.D., K. Barker, and C.M. Rocco. 2015. Stochastic Availability Importance Measures. In progress.
4. Barker, K. and J.E. Ramirez-Marquez. 2015. Availability-Based Importance Framework for Supplier Selection. *Proceedings of the Naval Postgraduate School Acquisition Research Symposium*, Monterrey, CA, May 2015.

Also, the following two theses were completed as a result of this work.

1. Robert D. Shaffer, Spring 2014, *Stochastic Modeling of Availability Importance Measures Using Monte Carlo Simulation*, MS Thesis, University of Oklahoma. Currently employed by OG&E.
2. Robert K. Hague, Spring 2014, *Interval-Valued Availability Framework for Supplier Selection*, MS Thesis, University of Oklahoma. Currently employed by Tinker Air Force Base.

TASK 1. AVAILABILITY-BASED IMPORTANCE MEASURES

This section is based on the following:
Gravette, M.A. and K. Barker. 2014. Achieved Availability Importance Measures for Enhancing Reliability Centered Maintenance Decisions. *Journal of Risk and Reliability*, **229**(1): 62-72.

1. Introduction and Motivation

The Department of Defense (DoD) uses three main metrics to measure the quality of one of its systems: reliability, maintainability, and availability [DoD 2005]. Particularly within the US Air Force, high quality aircraft equipment requires high performance values for all three metrics: reliable (ability to last as long as intended) and maintainable (ability to be fixed with minimum effort and time) to make the aircraft equipment available (accessible when needed). Availability, or the probability that a system is performing its required function at a given point in time when operated and maintained in a prescribed manner [Ebeling 2010], is perhaps the key metric of the three.

The availability of DoD systems is threatened by obsolescence. For example in the US Air Force, the cost to replace over 500 KC-135s, which debuted in the mid-1950s, has been estimated in the tens of billions of dollars with a replacement plan lasting for several decades [GAO 2004]. A budget reduction of about 29 percent since 1990 has "forced the branches of the military to extend the life of current legacy systems with significant reductions in new acquisitions of replacement systems" [Maithaisel 2008]. Such new purchases are usually restricted due to funding limitations, making redesigning or generating redundancy for improved reliability not an option [Kuo and Prasad 2000, Misra and Sharma 1973]. As such, the only remaining option to improve system availability is to enhance the maintenance methods during sustainment of the system. This leads to the need for an optimal maintenance policy to have the maximum positive impact on availability [Lie and Chun 1986, DoD 2011].

Recent DoD focus has been directed toward improving the decision making process for system sustainment, including maintenance, repair, and overhaul (MRO) operations and the acquisition of MRO parts. MRO depot resources must be dynamically assigned to reflect changes in priority driven by critical supply needs and internal parts shortages with the goal of reducing cost and lead times, meeting due dates, and maximizing availability of DoD weapon systems. Streamlined MRO activities, including the scheduling of system maintenance, the acquisition of parts (e.g., spare part shortages have been a concern in the DoD [Kuo and Zhu 2012]), and the optimal performance of supply chain operations, is key to keeping these aging systems available.

One means to tighten MRO costs is to focus on a primary set of components that most affect system performance. The analysis of systems, regardless of domain, often includes determining which system components are most influential on the performance of the system. Given the context above, the task of detecting the system, subsystem, or component on which to focus efforts (e.g., MRO activities) to gain the most improvement (e.g., improved availability) for the least cost is an important one. Component importance measures, a well-studied topic in reliability engineering [Kuo and Zhu 2012], measure the influence of particular components on the reliability of an overall system.

In a couple of recent theoretical exercises, importance measures have been developed to focus on availability metrics to determine the component in a system that most influences overall system availability [Cassady 2004, Barabady and Kumar 2007]. This paper extends this recent work by (i) analyzing availability-based component importance for some specific reliability and maintainability metrics that comprise *achieved availability*, an important DoD metric, and (ii)

proposing a simple decision making formulation to highlight how inspections can impact availability for those components found to be most impactful to the system. Section 2 provides some methodological background to the availability-based importance measures described here. Section 3 describes the importance measure for achieved availability, and Section 4 provides a DoD-inspired illustrative example. Section 5 closes with concluding remarks.

2. Methodological Background

Advancing technology and tighter budgets have prompted the desire to avoid failures before they occur, broadly referred to as preventative maintenance. There remained a desire to do more for less – more availability and reliability with focus on safety and environmental impacts while keeping budgets to a minimum – ultimately leading to the philosophy of Reliability-Centered Maintenance (RCM) [Moubray 1997]. Those who plan on using RCM do so because they expect to gain longer availability times, lower costs, and better control and decisions [Endrenyi et al. 2001]. With the idea of RCM in mind, we review background on calculations for component and system availability, as well as importance measures.

2.1. Availability Classification and Quantification

The reliability, availability, and maintainability performance metrics of a system have fundamental relationships. Given our interests in DoD weapons system in particular, we examine the similarity and differences among the DoD definitions of these system performance metrics [DoD 2005]:

- *Reliability* is the probability that an item can perform its intended function(s) without failure for a specified time under stated conditions.
- *Availability* is a measure of the degree to which an item is in an operable state and can be committed at the start of a mission when the mission is called for at an unknown (random) point in time.
- *Maintainability* is the probability that an item can be retained in, or restored to, a specified condition in a given time when maintenance is performed by personnel having specified skill levels, using prescribed procedures and resources, at each prescribed level of maintenance and repair.

Reliability is a metric that is often optimized during a system's design phase, where system configurations and component redundancies are considered to maximize system reliability or related mean time to failure metrics. However, after the design phase is completed, there is often no further action possible with the existing components to improve their reliability. As *reliability* as a metric does not account for maintenance considerations, *availability* is a more suitable metric to measure the effectiveness of an existing system. Furthermore, availability can be improved in an existing system through improved maintainability when the reliability values are not realized according to the manufacturer's specifications [Kuo and Wan 2006].

There are several definitions of availability depending on the user's point of view. All such definitions use Eq. (1) as a baseline, which essentially provides a probability of the system being operational. The various availability definitions differ is in how they define what is included in the *uptime* and *downtime* parameters. The three most common categories of availability are inherent availability, achieved availability, and operational availability [Lie et al. 1977].

$$\text{availability} = \frac{\text{uptime}}{\text{uptime} + \text{downtime}} \tag{1}$$

Inherent availability (A_i) is the most commonly used availability measurement. Inherent availability uses the component's or system's Mean Time Between Failures (MTBF) as the uptime measure and Mean Time To Repair (MTTR) the downtime measure. MTTR includes only corrective maintenance downtime. The A_i parameters initially come from the specifications in the manufacturer's report on how long the system is projected to operate before failure and how long it takes a normal maintenance team to repair a failed unit.

Operational Availability (A_o) is the best measure of the "realistic" availability a user of a system actually experiences over a period of time. This is because A_o is based on the collection of all of the actual events that occur to the system during any system downtime until the system is once again fully restored. It includes uptime as the Mean Time Between Maintenance (MTBM) actions and an additional term for ready time (RT), assuming the system is operational even if it is offline. The operational cycle is the total time being considered for the system. For the downtime parameter, A_o defines Mean Down Time (MDT) with an expanded definition of the time to repair the system plus the Delay Time (DT). The significant aspect of the MDT is that it includes administrative and logistic delays, while the system is down, and the system's Mean Maintenance Time (M) [Lie et al. 1977].

Achieved Availability (A_a) is the measure that the maintenance department would most often be tracking as a department performance measure of both the systems they maintain and the department capacity to maintain them, as it is based on both the actual maintenance touch time and the equipment's failure activity. The A_a definition and equation uses MTBM and M as its parameters. Achieved availability for a system with *n* components is calculated in Eq. (2) [Lie et al. 1977].

$$A_\text{a} = \frac{\text{MTBM}}{\text{MTBM} + \text{M}} \tag{2}$$

A_a includes both corrective maintenance actions, in the form of system failures, and preventative maintenance actions that take the system offline, in the form of *system downing PMs*.

$$\text{MTBM} = \frac{\text{Total uptime}}{\text{\# of system failures} + \text{\# of system downing PMs}} \tag{3}$$

$$\text{M} = \frac{\text{CM downtime} + \text{PM downtime}}{\text{\# of system failures} + \text{\# of system downing PMs}} \tag{4}$$

The weakness of A_i is that it does not include the PM parameter incorporated into the calculation. The weakness of A_o is that it is convoluted with many other logistics and administrative parameters and delays. The conclusion is that neither of these first two availability calculations are well suited for focusing on maintenance impacts to availability. It is the most

appropriate for this study as A_{a} incorporates PM and inspections can be split out of PM to analyze their core impact on A_{a}.

2.2. Importance Measures

It is very rare that systems are simplistic enough to have a minimum amount of components that would allow equal attention or worth to be given to all components. This is particularly true in the case of DoD weapons systems, which are often highly complex. Such systems lend themselves to allocate resources by dividing the system up into subsystems or collections of subsystems based on how important that subsystem is to the overall system. Then each subsystem can be more easily analyzed at the component level. The importance of each component is to the subsystem can better be measured, which then relates the component's importance to the overall aircraft.

Many component importance measures (CIMs) have been developed to determine the criticality of individual components to system performance [Miman and Pohl 2006]. Primarily, importance measures have been introduced to measure the influence of particular components on the overall reliability of the system [Kuo and Zuo 2003, Modarres et al. 2010]. Specific CIMs include risk reduction worth (RRW), an index that quantifies the potential damage to a system caused by a particular component, and the reliability achievement worth (RAW) of a component, or the maximum proportion increase in system reliability generated by that component [Ramirez-Marquez et al. 2006].

This work will focus on the Birnbaum importance measure, among the most widely used importance measures in reliability engineering [Fricks and Trivedi 2003]. For a system of *n* components, the Birnbaum importance measure [Birnbaum 1969] has historically measured how the change in reliability of component *i* influences a change in the reliability of the system, or $I_i^B = \partial R_s / \partial R_i$. After the I_i^B factor for each component is computed, the component with the largest I_i^B value is the component that will offer the greatest improvement in system reliability when its reliability is improved.

Both Cassady et al. [2004] and Barabady and Kumar [2012] adapted the I_i^B concept to propose an availability importance measure based on inherent availability. Their availability importance, in Eq. (5), measures demonstrate how the inherent availability of the system is influenced by the inherent availability of a subsystem or component. In their equations, A_s is the system availability and A_i is the component availability for component *i*.

$$I_i^A = \frac{\partial A_s}{\partial A_i} \tag{5}$$

Similar to Birnbaum reliability importance, the change in the component availability that will have the greatest impact on the reliability of the system is from the I_i^A with the largest value. This paper also extends the Birnbaum reliability importance measures to availability, but in this paper it is based on A_{a}, considered to be a much more appropriate maintenance centered availability measure. Further, we focus on importance relative to MTBM and M measures.

2.3. System Configurations and their Availability Measures

This paper addresses the four primary system configurations depicted in Figure 1: series, parallel, series-parallel, and parallel-series. And for these four system configurations, the notation for achieved availability is: $A_{\mathrm{a}}^{\mathrm{S}}$ for series systems, $A_{\mathrm{a}}^{\mathrm{P}}$ for parallel system, $A_{\mathrm{a}}^{\mathrm{SP}}$ for series-parallel, and $A_{\mathrm{a}}^{\mathrm{PS}}$ for parallel-series. For a series only combination of components, the subscript *i* represents any one of the *n* individual components in the series system. For parallel configurations, the subscript *j* refers to any one of the *m* individual parallel components of the parallel system or subsystem. For the series-parallel and parallel series combination, the achieved availability notation includes either "*ij*" or "*ji*" to represent the individual components of the respective systems. The achieved availability calculations for particular system configurations are variants of the general form in Eq. (2).

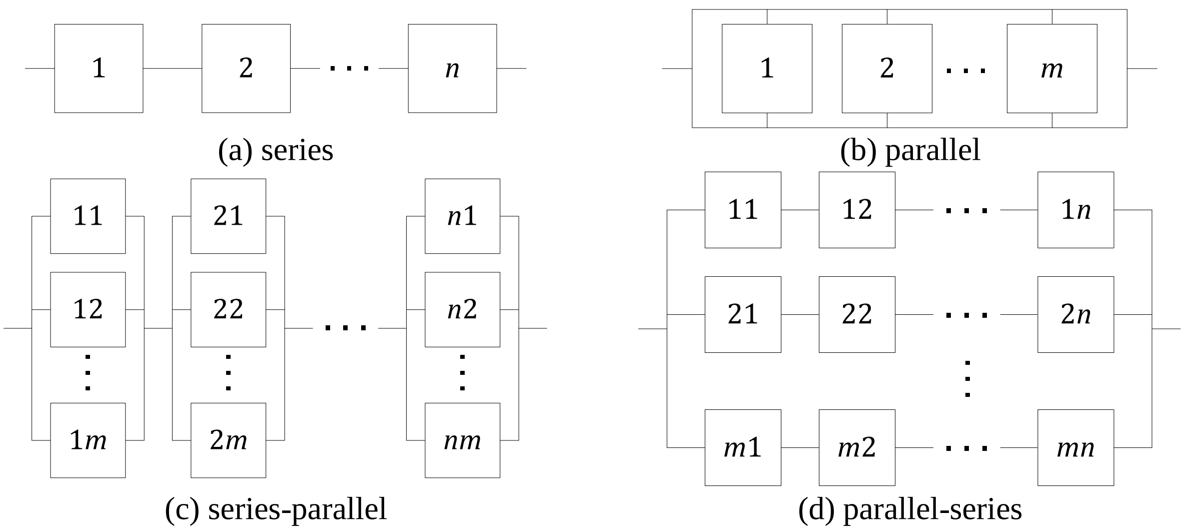

Figure 1. Four primary configurations that describe the structure of most systems.

2.4.1. Series Systems

Figure 1a depicts a system comprised of *n* independent components in series. For this system to be available, each independent component must be operable. The steady-state availability for a series system is the product of the independent component availabilities. Therefore, the system availability, A_{a}^{S} found in Eq. (6), will be smaller than the smallest component's availability [Ebeling 2010].

$$A_{\mathrm{a}}^{\mathrm{S}} = \prod_{i=1}^{n} A_{\mathrm{a}_i} = \prod_{i=1}^{n} \frac{\mathrm{MTBM}_i}{\mathrm{MTBM}_i + \mathrm{M}_i} \tag{6}$$

2.4.2. Parallel Systems

Figure 1b depicts a system comprised of *m* independent components connected in parallel. In order for this system to be unavailable, every independent component must be inoperable. This configuration can often be found when a system is critical to the operation of a weapons system

(e.g., safety of flight for an aircraft). The A_a^P is found in Eq. (7) as the probability that at least one component is not unavailable [Ebeling 2010].

$$A_{\mathrm{a}}^{\mathrm{P}} = \coprod_{j=1}^{m} A_{\mathrm{a}_j} = \coprod_{j=1}^{m} \frac{\mathrm{MTBM}_j}{\mathrm{MTBM}_j + \mathrm{M}_j} = 1 - \prod_{j=1}^{m} \left(1 - \frac{\mathrm{MTBM}_j}{\mathrm{MTBM}_j + \mathrm{M}_j}\right) \tag{7}$$

2.4.3. Series-Parallel Systems

A series-parallel system, depicted in Figure 1c, is comprised of *n* independent subsystems connected in series, where each subsystem consists of *m* components in parallel. This is one of the more complicated systems but is also among the more common situations for a high risk system (e.g., DoD aircraft). The redundancy allows some of the individual components to not be available and the system to still be available as long as at least one component in every subsystem is available. The achieved availability for this configuration is provided in Eq. (8).

$$A_{\mathrm{a}}^{\mathrm{SP}} = \prod_{i=1}^{n} \left[\coprod_{j=1}^{m} A_{\mathrm{a}_{ij}} \right] = \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m} \left(1 - \frac{\mathrm{MTBM}_{ij}}{\mathrm{MTBM}_{ij} + \mathrm{M}_{ij}}\right) \right] \tag{8}$$

2.4.4. Parallel-Series Systems

A parallel-series system, shown in Figure 1d, is a system comprised of *m* independent subsystems connected in parallel, where each subsystem consists of *n* components in a series configuration. A parallel-series system configuration allows for multiple components to not be available, as long as all the components in one of the subsystems are still available. Such a system configuration is especially useful in describing the subsystems of an aircraft system, illustrated subsequently in Section 4. The achieved availability in terms of the MTBM and M metrics of individual components is found in Eq. (9).

$$A_{\mathrm{a}}^{\mathrm{PS}} = \coprod_{j=1}^{m} \left(\prod_{i=1}^{n} A_{\mathrm{a}_{ji}} \right) = 1 - \prod_{j=1}^{m} \left(1 - \prod_{i=1}^{n} \frac{\mathrm{MTBM}_{ji}}{\mathrm{MTBM}_{ji} + \mathrm{M}_{ji}} \right) \tag{9}$$

3. Achieved Availability Importance Measures

Throughout this paper, independence is assumed for all the components of a system, suggesting that the failure of one component does not have an effect on the other components. However, depending on the configuration of the system, the system may be either in a failed or working state. Assumptions for all systems are as follows, adapted from [Cassady et al. 2004, Barabady and Kumar 2007]:

- The system is in steady state.
- Each system is comprised of independently and identically distributed (IID) components.
- All components are repairable, which returns the component to an as good as new state.
- Each component and system has two states: working or down, where down includes failures and down for maintenance.
- Components were designed to be easily accessible for maintenance.

This section extends the Birnbaum importance measure with three importance measures, motivated by the approaches of Cassady et al. [2004] and Barabady and Kumar [2012] but specifically (i) adapting achieved availability and (ii) constructing two new improvement measures based on MTBM and M. The first importance measure is the achieved availability importance measure for the component and can be calculated when *i* can equal *j*, *ij*, or *ji*, depending on the system configuration. Where A_{a} is the system achieved availability and A_{a_i} is the *i*th component's achieved availability, the achieved availability importance measure I_{a_i} is found in Eq. (10).

$$I_{\mathrm{a}_i} = \frac{\partial A_{\mathrm{a}}}{\partial A_{\mathrm{a}_i}} \tag{10}$$

The other two importance measures are based on the availability of the system, with respect to the MTBM and M parameters. These two importance measures are referred to here as *MTBM importance* and *M importance*.

The achieved availability importance measure with respect to the MTBM highlights how the mean time between maintenance, or a surrogate measure of reliability, of component *i* impacts the availability of the system. Provided in Eq. (11), the component with the largest value of $I_{\mathrm{a,MTBM}_i}$ indicates that it has the largest effect on the availability for the system.

$$I_{\mathrm{a,MTBM}_i} = \frac{\partial A_{\mathrm{a}}}{\partial \mathrm{MTBM}_i} = \frac{\partial A_{\mathrm{a}}}{\partial A_{\mathrm{a}_i}} \times \frac{\partial A_{\mathrm{a}_i}}{\partial \mathrm{MTBM}_i} \tag{11}$$

The M importance measure highlights how the mean maintenance time of component, *i*, impacts the availability of the entire system. The M of a component is a surrogate measures describing its maintainability. When evaluating Eq. (12), the $I_{\mathrm{a,M}_i}$ resulting values are negative (as smaller values of M result in larger values of system availability). Since the goal of the importance component process is to find the component that has the largest $I_{\mathrm{a,M}_i}$ magnitude to determine the speed of which it changes, the negative sign of the results will be ignored and all of the results are provided as absolute values to enhance chart and graph comparisons of values.

$$I_{\mathrm{a,M}_i} = \frac{\partial A_{\mathrm{a}}}{\partial \mathrm{M}_i} = \frac{\partial A_{\mathrm{a}}}{\partial A_{\mathrm{a}_i}} \times \frac{\partial A_{\mathrm{a}_i}}{\partial \mathrm{M}_i} \tag{12}$$

Explicitly highlighting the contributions of MTBM and M in Eq. (5) allows us to pinpoint which component of availability to concentrate on for the most important components: improve reliability (e.g., through supplier selection) or improve maintainability (e.g., through MRO on-hand inventory).

Characteristically, the MTBM is a much greater value than the M. When that is the case, $I_{\mathrm{a,M}_i}$ will be much greater than $I_{\mathrm{a,MTBM}_i}$ suggesting that finding ways to decrease the component's M offers a greater benefit to the system availability then working on the component's MTBM. However, the cost and effort required to decrease M may be more significant than it is required

to increase the MTBM value. Depending on the MTBM and M values, the reverse situation could also be possible as well. To be certain the initial maintenance focus is on the correct parameter as determined by these additional importance measures both the $I_{\mathrm{a,MTBM}_i}$ and the $I_{\mathrm{a,M}_i}$ should always be calculated.

3.1. A_a Importance Measures for a Series System

From Eq. (6), the achieved availability importance measure is found in Eq. (13) for a series system.

$$I_{\mathrm{a}_i}^{\mathrm{S}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{S}}}{\partial A_{\mathrm{a}_i}} = \prod_{k \neq i}^{n} A_{\mathrm{a}_k} \tag{13}$$

The MTBM importance and M importance are computed for the target component in the series system, shown in Eqs. (14) and (15), respectively.

$$I_{\mathrm{a,MTBM}_i}^{\mathrm{S}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{S}}}{\partial A_{\mathrm{a}_i}} \times \frac{\partial A_{\mathrm{a}_i}}{\partial \mathrm{MTBM}_i} = I_{\mathrm{a}_i}^{\mathrm{S}} \times \frac{\partial A_{\mathrm{a}_i}}{\partial \mathrm{MTBM}_i} = A_{\mathrm{a}}^{\mathrm{S}} \times \frac{\mathrm{M}_i}{\mathrm{MTBM}_i(\mathrm{MTBM}_i + \mathrm{M}_i)} \tag{14}$$

$$I_{\mathrm{a,M}_i}^{\mathrm{S}} = I_{\mathrm{a}_i}^{\mathrm{S}} \times \frac{\partial A_{\mathrm{a}_i}}{\partial \mathrm{M}_i} = \frac{\partial A_{\mathrm{a}}^{\mathrm{S}}}{\partial A_{\mathrm{a}_i}} \times \frac{\partial A_{\mathrm{a}_i}}{\partial \mathrm{M}_i} = A_{\mathrm{a}}^{\mathrm{S}} \times \frac{1}{(\mathrm{MTBM}_i + \mathrm{M}_i)} \tag{15}$$

3.2. A_a Importance Measures for a Parallel System

From the definition of parallel system achieved availability in Eq. (7), the associated importance measure, $I_{\mathrm{a}_j}^{\mathrm{P}}$, is provided in Eq. (16).

$$I_{\mathrm{a}_j}^{\mathrm{P}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{P}}}{\partial A_{\mathrm{a}_j}} = 1 - \prod_{l \neq j}^{m} (1 - A_{\mathrm{a},l}) \tag{16}$$

With Eqs. (17) and (18) respectively, the MTBM and M importance measures can be further computed for the target component in a parallel system.

$$I_{\mathrm{a,MTBM}_j}^{\mathrm{P}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{P}}}{\partial A_{\mathrm{a}_j}} \times \frac{\partial A_{\mathrm{a}_j}}{\partial \mathrm{MTBM}_j} = \left[1 - \prod_{l \neq j}^{m} \left(1 - A_{\mathrm{a}_l}\right)\right] \times \frac{\mathrm{M}_j}{\mathrm{MTBM}_j(\mathrm{MTBM}_\mathrm{j} + \mathrm{M}_j)} \times A_{\mathrm{a}_j} \tag{17}$$

$$I_{\mathrm{a,M}_j}^{\mathrm{P}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{P}}}{\partial A_{\mathrm{a}_j}} \times \frac{\partial A_{\mathrm{a}_j}}{\partial \mathrm{M}_j} = \left[1 - \prod_{l \neq j}^{m} \left(1 - A_{\mathrm{a}_l}\right)\right] \times \frac{1}{(\mathrm{MTBM}_j + \mathrm{M}_j)} \times A_{\mathrm{a}_j} \tag{18}$$

3.3. A_a Importance Measures for a Series-Parallel System

Eq. (19) illustrates the impact of the achieved availability *ij*th component on the system achieved availability for a series-parallel system. Priority of ranking of which component to start with, in terms of best choice for increasing the system availability, should be assigned to component *ij* with the maximum $I_{\mathrm{a}_{ij}}^{\mathrm{SP}}$. Focus is given to the impact of MTBM and M to system achieved availability with Eqs. (20) and (21).

$$I_{\mathrm{a}_{ij}}^{\mathrm{SP}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{SP}}}{\partial A_{\mathrm{a}_{ij}}} = \prod_{k \neq i}^{n} \left[1 - \prod_{l=1}^{m} \left(1 - A_{\mathrm{a}_{kl}} \right) \right] \times \prod_{l \neq j}^{m} \left(1 - A_{\mathrm{a}_{il}} \right) \tag{19}$$

$$I_{\mathrm{a,MTBM}_{ij}}^{\mathrm{SP}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{SP}}}{\partial A_{\mathrm{a}_{ij}}} \times \frac{\partial A_{\mathrm{a}_{ij}}}{\partial \mathrm{MTBM_{ij}}} = I_{\mathrm{a}_{ij}}^{\mathrm{SP}} \times \frac{\mathrm{M}_{ij}}{\mathrm{MTBM}_{ij}(\mathrm{MTBM}_{ij} + \mathrm{M}_{ij})} \times A_{\mathrm{a}_{ij}} \tag{20}$$

$$I_{\mathrm{a,M}_{ij}}^{\mathrm{SP}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{SP}}}{\partial A_{\mathrm{a}_{ij}}} \times \frac{\partial A_{\mathrm{a}_{ij}}}{\partial \mathrm{M}_{ij}} = I_{\mathrm{a}_{ij}}^{\mathrm{SP}} \times \frac{1}{(\mathrm{MTBM}_{ij} + \mathrm{M}_{ij})} \times A_{\mathrm{a}_{ij}} \tag{21}$$

3.4. A_a Importance Measures for a Parallel-Series System

From the achieved availability calculation for a parallel-series system in Eq. (9), the importance measure for component *ji* achieved availability is provided in Eq. (22). is used to compute the achieved availability importance measures for a parallel-series system.

$$I_{\mathrm{a}_{ji}}^{\mathrm{PS}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{PS}}}{\partial A_{\mathrm{a}_{ji}}} = \prod_{l \neq i}^{m} \left[1 - \prod_{k=j}^{n} A_{\mathrm{a}_{lk}} \right] \times \prod_{k \neq j}^{n} A_{\mathrm{a}_{jk}} \tag{22}$$

The MTBM and M importance measures are computed for the target component in a parallel-series system in Eqs. (23) and (24).

$$I_{\mathrm{a,MTBM}_{ji}}^{\mathrm{PS}} = \frac{\partial A_{\mathrm{a}}^{\mathrm{PS}}}{\partial A_{\mathrm{a}_{ji}}} \times \frac{\partial A_{\mathrm{a}_{ji}}}{\partial \mathrm{MTBM}_{ji}} = I_{\mathrm{a}_{ji}}^{\mathrm{PS}} \times \frac{\mathrm{M}_{ji}}{\mathrm{MTBM}_{ji}(\mathrm{MTBM}_{ji} + \mathrm{M}_{ji})} \times A_{\mathrm{a}_{ji}} \tag{23}$$

$$I_{\mathrm{a,M}_{ji}}^{\mathrm{PS}} = \frac{\partial \mathrm{A}_{\mathrm{a}}^{\mathrm{PS}}}{\partial A_{\mathrm{a}_{ji}}} \times \frac{\partial A_{\mathrm{a}_{ji}}}{\partial \mathrm{M}_{ji}} = I_{\mathrm{a}_{ji}}^{\mathrm{PS}} \times \frac{1}{(\mathrm{MTBM}_{ji} + \mathrm{M}_{ji})} \times A_{\mathrm{a}_{ji}} \tag{24}$$

4. Illustrative Examples: Modeling the Impact of Inspections

An inspection decision making framework motivated by achieved availability importance measures is provided in this section, wherein we identify the components that have the largest magnitude importance measure and also maximize the system achieved availability for each system by allowing the adjustment of the number of inspections for each component. The four system configurations are illustrated with an example inspired by the components of an aircraft.

4.1. A_a Importance Measure Decision Making Framework

Often MTBM and M measures do not differentiate between preventive maintenance actions (the physical activity required to performance PM) and inspection actions (which may or may not result in PM actions depending on the result of the inspection). As a primary objective of this IM-inspired framework is to assist in determining the frequency of inspections, the difference between PM and inspection should be explicit. As such, the calculations of MTBM in Eq. (3) and M in Eq. (4) are modified with Eqs. (25) and (26) by separating out the parameter "# of Inspections" in the denominators of both equations and the parameter "Inspection Time" in the numerator of Eq. (26).

$$\mathrm{MTBM} = \frac{\text{Total uptime}}{\text{\# of CMs} + \text{\# of PMs} + \text{\# of Inspections}} \tag{25}$$

$$\mathrm{M} = \frac{\text{CM downtime} + \text{PM downtime} + \text{Inspection time}}{\text{\# of CMs} + \text{\# of PMs} + \text{\# of Inspections}} \tag{26}$$

Recall the basic availability formula in Eq. (1), the ratio of *uptime* to *uptime + downtime*. To maximize availability, either uptime has to be maximized or downtime has to be minimized. Downtime, the numerator in Eq. (26), can be expressed with Eq. (27), where n_{CM}, n_{PM}, and n_{I} refer to the numbers of CM, PM, and inspection actions, respectively, and y_{CM}, y_{PM}, and y_{I} refer to the mean downtime for those actions. The total downtime for CM is $n_{\mathrm{CM}} y_{\mathrm{CM}}$, the total downtime for PM is $n_{\mathrm{PM}} y_{\mathrm{PM}}$, and the total downtime for the inspection is $n_{\mathrm{I}} y_{\mathrm{I}}$.

$$\text{downtime} = n_{\mathrm{CM}} y_{\mathrm{CM}} + n_{\mathrm{PM}} y_{\mathrm{PM}} + n_{\mathrm{I}} y_{\mathrm{I}} \tag{27}$$

For those components deemed important to availability by the importance measures above, particularly those important to the mean downtime metric, M, the optimal number of inspections to improve availability is determined with the conceptual optimization problem found in Eq. (28) for component *i*. To accommodate for inspection-related decisions, the CM downtime and PM downtime calculations are modified. Two new parameters are introduced: P is the number of inspections before a replacement is done, and C is a proportional multiplier to be applied to the repair time to determine the replacement time. In typical maintenance environments, all variables would be set values for Eq. (28) and assumed for a steady state system.

$$\min_{n_{\mathrm{I}_i}} \left(n_{\mathrm{CM}_i} - \frac{n_{\mathrm{I}_i}}{P}\right) y_{\mathrm{CM}_i} + \left(C y_{\mathrm{CM}_i} - \frac{n_{\mathrm{I}_i}}{P}\right) + \left(n_{\mathrm{PM}_i} - \frac{n_{\mathrm{I}_i}}{P}\right) y_{\mathrm{PM}_i} + \left(n_{\mathrm{I}_i} y_{\mathrm{I}_i}\right)$$
$$\text{s.t. } n_{\mathrm{I}_i} \geq 0 \tag{28}$$

$$\text{all other parameters } n_{\mathrm{CM}_i}, n_{\mathrm{PM}_i}, y_{\mathrm{CM}_i}, y_{\mathrm{PM}_i}, y_{\mathrm{I}_i} \geq 0,\ 0 \leq C \leq 1,\ 0 \leq P \leq 1$$

The ranking of components according to their availability importance measures produces a lexicographically ordered maximum value set, of which *i* is a member. The optimization problem in Eq. (28) could be performed for multiple components in this set, or an optimization problem could be designed to account for the inspection of multiple components simultaneously if applicable.

4.2. Illustrative Example Background: Aircraft Maintenance

Field aircraft maintenance data were acquired to illustrate the application of all of the different example systems by applying the data in the decision making framework for the different achieved availability importance measures to each appropriate system. Note that a random factor was applied to the raw data to mask the aircraft from which the data were derived and also the component names are not used. However, proportional relationships among components and the overall performance parameters are still reasonably characteristic in the resulting component priority predictions made by the component importance measure determinations.

The data were collected for a parallel-series system that is comprised of a parallel system made up of four subsystems, with each subsystem a series configuration with three components. The individual component names are referred to as Component 11, Component 12,…, Component *ji* because the actual configuration is in parallel-series. This scenario is depicted in Figure 2.

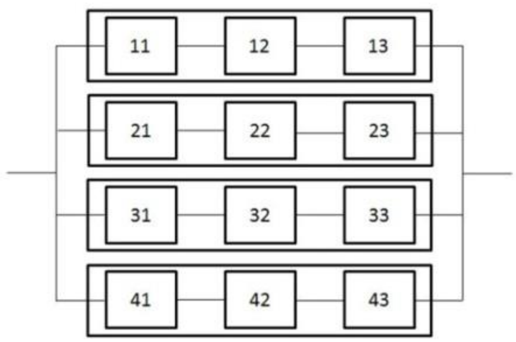

Figure 2. Diagram of the parallel-series aircraft system configuration.

Data were collected for a one year period (potential maximum total uptime time of 8,760 hours) from field aircraft and is used for the illustrating and analysis sections of this paper. The data representing the variables in Eq. (28), shown in Table 1, include the number of and downtime length for CM actions, the number of and downtime length for PM actions, and the number of and downtime length for inspections.

Table 1. CM, PM, and inspection data for the aircraft system.

Component	# of CM down	Time per CM	# of PM down	Time per PM	# of inspections	Time per inspection
ji	n_{CM}	y_{CM}	n_{PM}	y_{PM}	n_{I}	y_{I}
11	10	1.59	46	3.66	7	9.78
12	12	13.31	41	4.03	7	9.78
13	19	2.81	53	4.09	7	9.78
21	22	2.88	60	5.58	9	8.16
22	15	4.63	80	6.76	5	1.47

23	20	5.27	66	9.67	5	1.70
31	20	5.74	75	6.60	7	3.95
32	10	5.10	73	5.94	7	2.13
33	15	43.01	29	15.13	3	24.14
41	10	37.74	19	15.04	3	8.93
42	14	47.47	26	14.59	5	19.04
43	10	53.47	19	16.91	3	24.14

The inspection that provides the insight to replace the component before it fails is called the "critical" inspection. The actual number of inspections needed before the critical inspection occurs would be determined by the historical inspection data and communicating with maintenance analysts. It is assumed that the number of inspections added before the critical inspection yields an accurate failure prediction is a fully adjustable parameter for each component.

The amount of time that a replacement takes is also a parameter in the decision making framework to allow variation of different component experiences. When a component is replaced, as determined by the critical inspection, the mean replacement time is used in place of the mean failure time and the CM and PM counts are reduced by one event. This assumption is realistic because after the replacement of a component, CM and PM would not be performed on the brand new equipment.

Subsets of this aircraft illustrative example will be used to illustrate a series subsystem configuration and the actual parallel-series configuration of Figure 2.

4.3. A_a Framework Example: Series System

The data chosen from Table 1 for the series system describe components 11, 12, and 13, comprising the first of four series subsystem.

Figure 3 depicts each of the individual component availabilities and the system availability, all shaded in gray. The $I_{\mathrm{a}_i}^{S}$ for each component shown in black. Note the different vertical axes: the trend of the relationship between the two calculations is of greater interest. As expected for a series system, the component with the least availability is the component with the largest system importance measures. From the calculation of the importance measures, it is seen that Component 13 becomes the clear choice with the highest priority component to focus improvement efforts: the more available Component 13 becomes, the more available the system becomes.

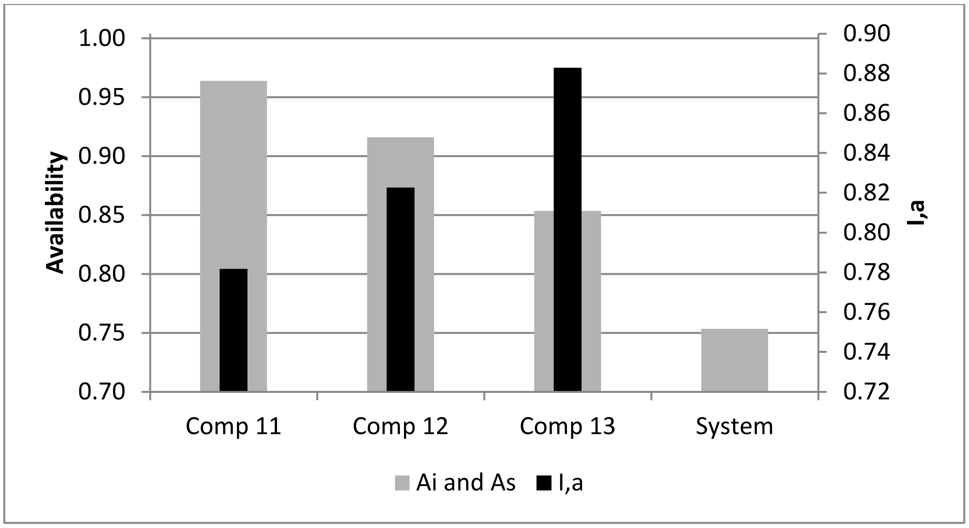

Figure 3. Availability and Aa importance for components in the series subsystem.

Now that Component 13 has been identified, the next step is to make a comparison between $I^S_{\mathrm{a,MTBM}_i}$ and $I^S_{\mathrm{a,M}_i}$ for Component 13 to further determine which of these parameters can provide the greatest positive impact on the system availability. This comparison is shown in Figure 4, which depicts that the $I^S_{\mathrm{a,M}_i}$ is greater compared to $I^S_{\mathrm{a,MTBM}_i}$. This suggests that improving M, or improving the maintainability of the component, for Component 13 provides most improvement for system availability.

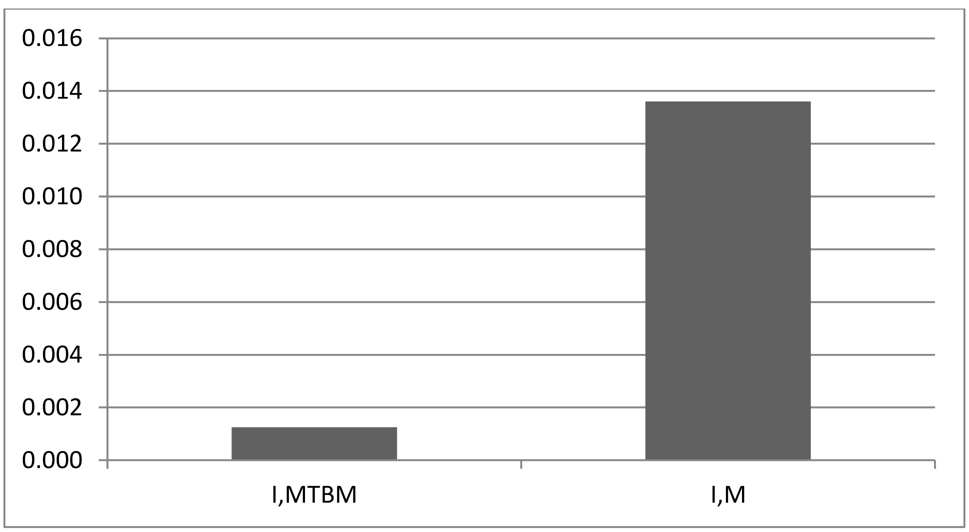

Figure 4. Comparing MTBM and M importance for Component 13.

Given that Component 13 and the MTBM parameter have been identified as having most impact on system availability, Eq. (28) is deployed in a discrete form to determine the number of inspections for each component (taken individually) to further influence system availability. Figure 5 provides something of a sensitivity analysis, varying each component individually while increasing the number of inspections to visualize the impact to the system availability. Figure 5

shows that, indeed, Component 13 has the largest, fastest, and greatest impact on the system. When the inspections are increased for Component 13, it provides the greatest opportunity to increase system availability up to 14 inspections. After that, the downtime attributed to inspections outweighs the downtime due to potential corrective maintenance actions.

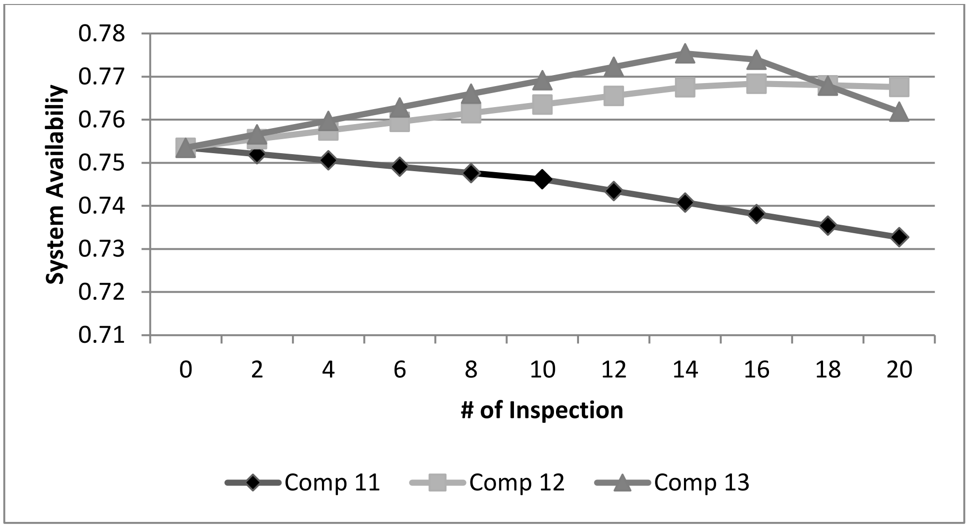

Figure 5. Series subsystem Aa sensitivity analysis.

Notice, Component 11 actually shows a negative impact on the system availability while increasing inspections. This is due to the inspection time for Component 11 being significantly longer than the replace or repair time. This points out an area for a maintenance process improvement, as an inspection likely should not take such an amount of time that the system cannot benefit from an inspection.

All of these steps and analysis can now be repeated for the remainder of the system configurations. What was intuitively predictable for a series system without the use of importance measure computations becomes untenable for the more complex configurations, only with the computations, charts, and graphs does it become immediately possible to make the same type of results and optimal predictions for the systems.

4.4. A_a Framework Example: Parallel-Series System

A series-parallel illustration is not provided, as the example in Figure 2 does not readily lend itself to such an application. Focus is given to the actual configuration in Figure 2, the parallel-series system, which is not featured in the work of Barabady and Kumar [2012]. No changes to the structure of the data in Table 1 were made for this illustration.

Figure 6 shows the individual component availabilities and system availability in gray, with the $I_{\mathrm{a}_{ji}}^{\mathrm{PS}}$ measures for each component in black. The component with the largest $I_{\mathrm{a}_{ji}}^{\mathrm{PS}}$, is shown as Component 13, which is prioritized as the first to focus improvement efforts. However, there are other components with $I_{\mathrm{a}_{ji}}^{\mathrm{PS}}$ values that are nearly the same magnitude and could also be under consideration for improvement, including Components 43 and 33. These components should be ranked in priority, and other extraneous factors not chosen in this analysis could change the

lexicographic order of these highly ranked components. The following analysis will focus on the highest ranked, Component 13.

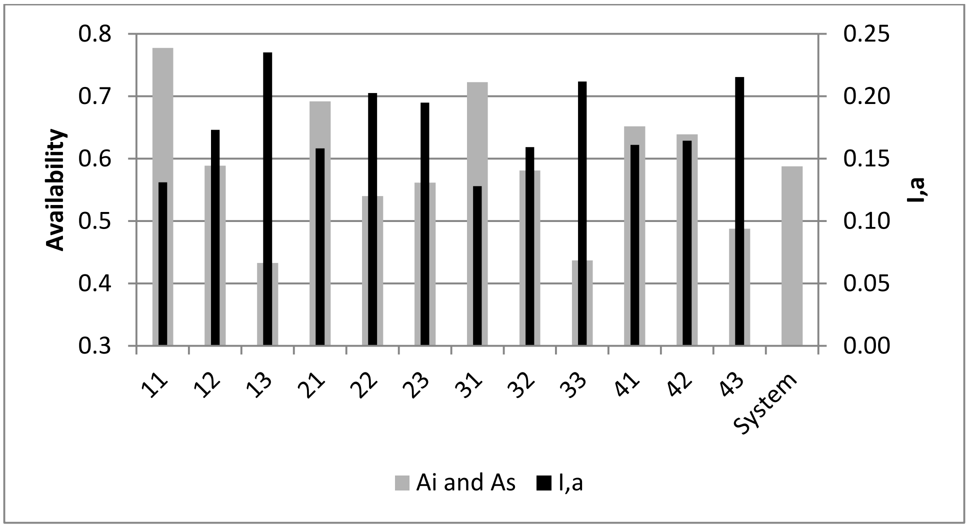

Figure 6. Availability and Aa importance for components in the entire parallel-series system.

A comparison for Component 13 between $I^{\mathrm{PS}}_{\mathrm{a,MTBM}_{ji}}$ and $I^{\mathrm{PS}}_{\mathrm{a,M}_{ji}}$ further determines which parameter can have the greatest impact on the system. This comparison is done in the chart in Figure 7, which suggests Component 13 shows the $I^{\mathrm{PS}}_{\mathrm{a,MTBM}_{ji}}$ is the one with the larger magnitude. Thus, an improvement on the MTBM for Component 13 would be expected to yield a greater impact on the system availability with the less effort. This implies that the system is down more them than it is up. Consider looking into a potential issue with the reliability causing extremely high failures or the maintenance techniques are taking an excessive amount of time.

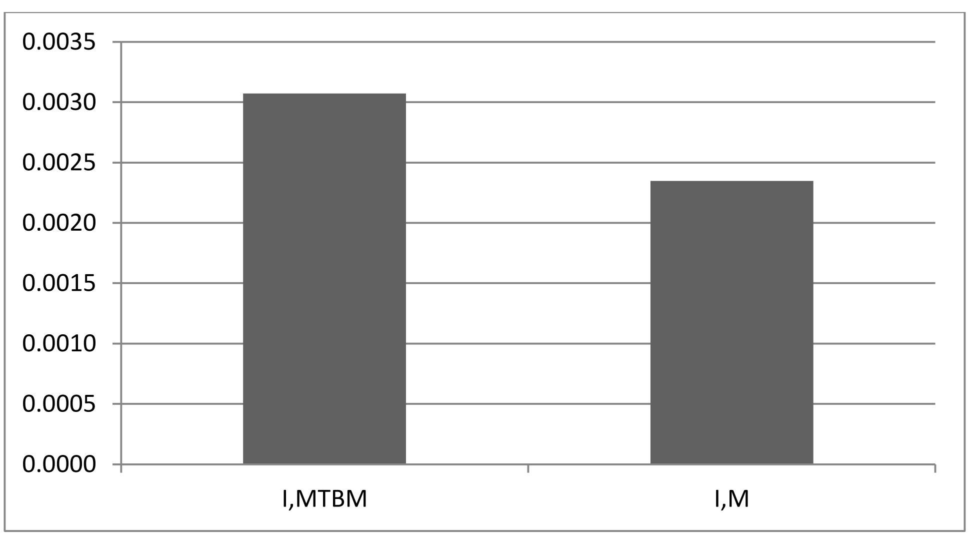

Figure 7. Comparing MTBM and M importance for Component 13.

Figure 8 provides the sensitivity implementation of Eq. (28) examining each component individually while increasing the inspections to see the impact to system availability.

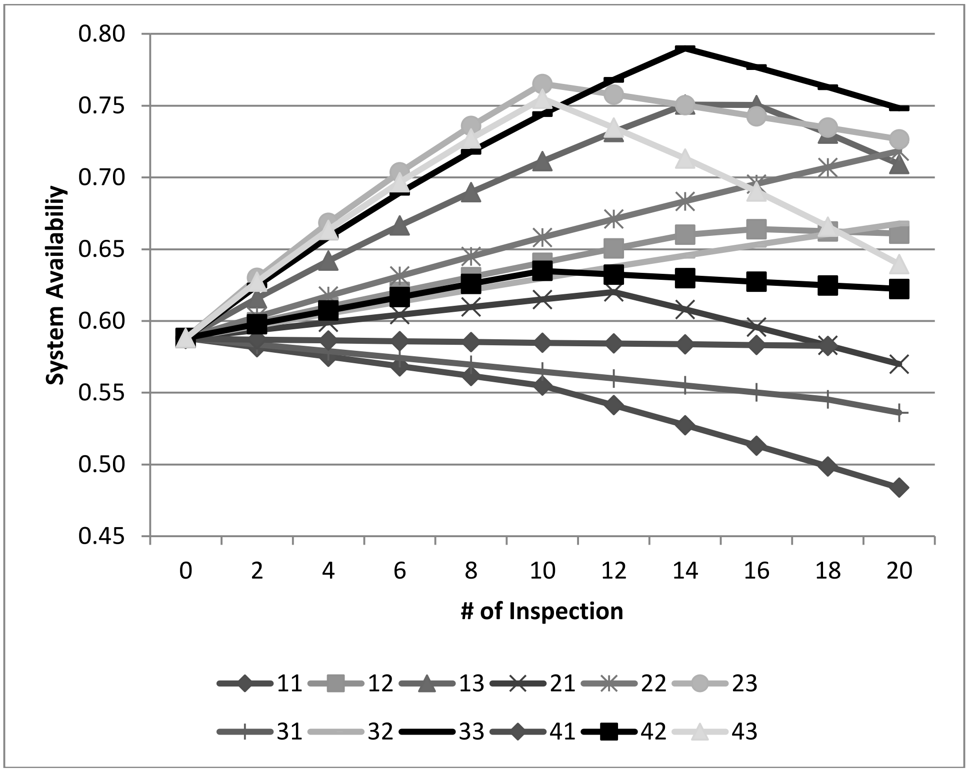

Figure 8. Parallel-series system Aa sensitivity analysis.

Figure 8 shows that Component 33 reaches the highest system achieved availability with critical inspections at inspection 14. Component 23 is the second highest, but takes fewer added inspections to reach its maximum impact to the system. Figure 8 indicates the third component for each subsystem (Component 13, 23, 33, and 43) as the components that can impact the system achieved availability the greatest with critical inspections added. This has implications for the particular aircraft example.

5. Concluding Remarks

This paper a means to (i) extend existing availability importance measures to include the effects of maintenance activity, (ii) determine the number of inspections to obtain the maximum system achieved availability, and (iii) apply this new availability importance measure tied to the optimal inspection frequency within an RCM decision making framework.

The availability-based framework has proven to be very useful to allow data to drive decisions when improving the achieved availability of a sample system. The importance measure is a deterministic objective measure that reports the status of the component, in particular, using achieved availability as its basis measure. It then points to the component that has the most opportunity for improvement, further identifying whether the MTBM or M parameter needs the most work for the system availability to improve. Each result is dependent upon the status of all

the components in the system and identifies directly to the one that needs improvement. Otherwise, time and money can easily be wasted by putting efforts into something that could result with a reduced system achieved availability.

An ideal realm to execute the findings of the decision making framework is RCM. The main objective of RCM is to be used for determining optimal failure management tactics, to include maintenance policies and provide proof for results from maintenance tasks. The illustrative decision making framework results assist by allowing data to efficiently and effectively drive decisions to accomplish the RCM objective.

Further work lies in applying a simulation-based approach to determining the availability-based importance of the components in more complex structures, paralleling the analytical approach in this paper.

Task 2. Stochastic Availability Importance Measures

This section is based on the following:
Shaffer, R.D., K. Barker, and C.M. Rocco. 2015. Stochastic Availability Importance Measures. In progress.

1. Introduction

In any organization, a system of systems exists in order to produce a product from various material and mental inputs. These systems can be broken down into sub components all the way to where the actual machines perform the work. At the machine and component level, various techniques are applied to optimize machine effectiveness. One technique used to optimize a system is the availability importance measure of a component within a system.

As defined by Cassady, Pohl and Song [2004] "a repairable system is defined as a system which, after failure, can be restored to an operating (functioning) condition by some maintenance action other than replacing the entire system." Replacing an entire system is a possibility, but not always in the best interests of the organization intent on proper resource allocation. The intent of the maintenance organization is to provide the production group (the customer or user of the equipment) with an asset that is available when needed in a cost effective manner [Gulati 2009].

In performing maintenance on equipment, several methods exist in which an organization can expend resources to restore functionality to faulty equipment. Usually the maintenance action is meant to restore the equipment back to a state defined as "good as new." A complete accurate repair provides the equipment the ability to perform its intended function as needed on a customer's schedule. This requires high availability and reliability of the systems [Gulati 2009]. Equipment that receives maintenance can expect to have several categories of maintenance performed on it. These are Corrective Maintenance (CM), Preventative Maintenance (PM) or inspections, Predictive Maintenance (PdM) and Condition Based Maintenance (CBM) which are all aspects of a reliability centered maintenance paradigm. Sometimes the maintenance involves a rebuild of a component or a complete sub-system replacement. Fortunately, today's manufacturing designs have created equipment as a system of components with items that can be replaced as they fail or approaching a state of failure.

It is unfortunate when highly critical assets experience a failure, but it is disastrous when these equipment failures cause some of the worst accidents and environmental damages in human history such as the March 2011 Fukushima Nuclear Power Plant Explosions to aviation disasters. In response to these types of events, there has existed for 40 years, a strategic framework for ensuring that equipment continues to perform as required [Moubray 1997]. This framework is known as Reliability Centered Maintenance (RCM) and contains the basis for measuring and using availability, a mathematical result of both uptime and downtime.

The motivation of this article is to explore the use of Availability Importance Measures (AIM) given as a function of random variables and using a stochastic approach such as Monte Carlo Simulation (MCS) to better determine a system(s) or component's importance ranking within the

system. Furthermore, a comparison between the Traditional Point Estimate (TPE) approach in common use today along with the Monte Carlo Simulation (MCS) methodology of using random variable probability distributions to determine if there is a difference with the chosen component or system importance.

Understanding the correct system or component to target for further availability improvement may ensure the organization focuses its resources efficiently and achieves its goals of higher productivity, higher quality production and assets that are ready and available as required.

2. Methodological Background

2.1. Appropriate Measure for Effectiveness, Availability

Availability is used as a measure of a system's reliability as well as inherent ability to account for the maintainability of an asset [Lie 1977] and is defined mathematically as:

$$\text{Availability} = \frac{\text{Uptime}}{\text{Uptime} + \text{Downtime}} \tag{29}$$

Where uptime is the amount of time a piece of equipment, system or component is ready to perform work and downtime is the amount of time required to maintain that system or component. The summation of uptime and downtime is usually the total time the system is possibly available but this is not necessarily equal to calendar time. For instance, a two shift production shift operating for eight hours in each day for five days a week will consume eighty hours of the available 168 hours in a calendar week. This can have serious implications for determining availability's component of downtime if a proactive maintenance action is performed on a non-production portion of the day, i.e. the remaining 8 hours in a day. So for simplicity, downtime will only include the unscheduled maintenance of a component and the uptime will include only as much as the associated shift time scheduled. This will closely resemble what is known as inherent availability, A_i, as defined as:

$$A\text{vailability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{30}$$

Where MTBF is the Mean Time Between Failure and MTTR is the Mean Time To Repair as defined below:

$$\text{MTBF} = \frac{\text{Uptime}}{\text{Number of System or Component Failures}} \tag{31}$$

$$\text{MTTR} = \frac{\text{Corrective Maintenance Downtime}}{\text{Number of System or Component Failures}} \tag{32}$$

Using availability as defined in any of the above equations, a component or system can be improved through either affecting the uptime or design of the component or through the

downtime or the maintainability of the component via proactive maintenance practices and precision installation practices that improve upon the time involved in repair [Kuo et al. 2006]. For simplicity, focus will be on availability simply as the relationship between uptime and downtime.

To define the overall importance of a component, a derivation of Birnbaum's Reliability importance measure used and is defined as:

$$\mathrm{I}_i^B = \frac{\partial R_s(t)}{\partial R_i(t)} \tag{33}$$

Where I_i^B = Birnbaum importance of the i^{th} component
$R_s(t)$ = System Reliability at time t
$R_i(t)$ = Component i reliability at time t
For $i = 1, 2\ldots n$
For n components

The component with the largest Birnbaum Reliability signifies that component will have the greatest overall system improvement when its reliability is re-engineered.

2.2. Derived Availability Importance

As referenced in Barabady and Kumar [2012]:

> "*Availability and reliability are good evaluations of a system's performance. Their values depend on the system structure as well as the component availability and reliability. These values decrease as the component ages increase; i.e. their serving times are influenced by their interactions with each other, the applied maintenance policy and their environments [Samrout et al. 2005]. The main requirements for the operation of complex systems are usually specified in terms of cost and availability and/or reliability, or equivalently in terms of mean time between failures and/or mean time to repair under a cost constraint*."

Transforming the reliability importance measure to the concept of availability importance measures, allows for the inclusion of maintenance actions and the idea that some of the components could be more important than other. This is then used this to quantify the key components as most sensitive to the system's overall availability. This enables the weakest areas of a system to be identified in both terms of maintainability and reliability [Beeson and Andrews 2003]. Understanding the availability importance, resources can be best allocated to the correct system or component of the system that rank highest in importance.

The work done by Cassady et al. [2004] and Barabady and Kumar [2012] successfully derived the availability importance measure from the Birnbaum Reliability importance measure. Creating the availability importance measure acknowledges the role uptime and downtime perform in a system or component. Final derivation of the availability importance measure for the i^{th} component is shown below:

$$I_i^A = \frac{\partial A_s(t)}{\partial A_i(t)} \tag{34}$$

Where I_i^A = Availability importance of the i^{th} component
$A_s(t)$ = System availability at time t
$A_i(t)$ = Component availability at time t
For all i = 1, 2… n
For n components

The availability importance component that has the largest value signifies the component that has the greatest positive effect on system availability when it is properly re-engineered.
To use the availability importance measure, an understanding of the system configuration and design are required. Systems can be arranged in a combinatorial effect of series and or parallel components with a specific equation to describe therein.

Systems can be designed various ways from a simple series (S) or parallel (P) configuration to a combination of parallel-series (PS) or series-parallel (SP) each with their own unique effects on overall system availability and reliability. Understanding the configuration of the system or components has a large impact on how the systems availability resolves and how the availability importance measure of each component affects that system.

2.3.1. Series System

For a simple series system, as shown in Figure 9. Series System, each component must be functional in order for the entire system to perform. Therefore, the system as a whole is dependent on its lowest availability component.

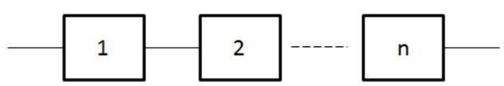

Figure 9. Series System

The availability importance measure for a Series System is shown below.

$$I_i^S = \frac{\partial A^S}{\partial A_i} = \prod_{k \neq i}^{n} A_k \tag{35}$$

2.3.2. Parallel System

For a simple parallel system, as shown in Figure 10. Parallel System, only one component must be functional for the system to perform. This is known as a redundant system. Therefore, the system availability as whole is equal to the component with the highest availability.

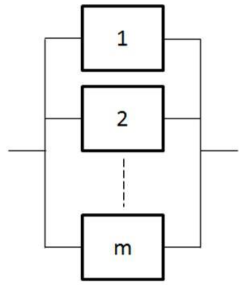

Figure 10. Parallel System

The availability importance measure for a Parallel System is as follows.

$$I_j^{\mathrm{P}} = \frac{\partial A^{\mathrm{P}}}{\partial A_j} = 1 - \prod_{l \neq j}^{m} (1 - A_l) \tag{36}$$

2.3.3. Series Parallel System

For a system such as Series Parallel, as shown in Figure 11, there are n sub components in parallel with m sub-systems in series. This type of overall system allows for built in redundancy at the n sub-system level, however, determination of the most important component becomes fairly complicated.

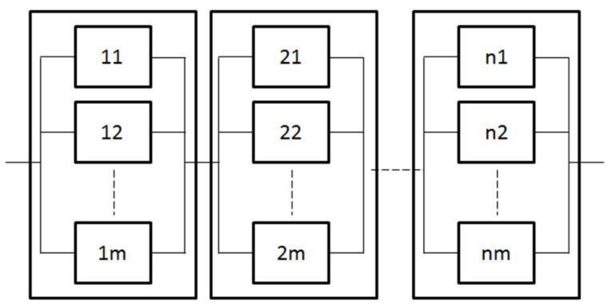

Figure 11. Series Parallel (SP) System

The derived availability importance of a Series Parallel (SP) system is shown as follows [Gravette and Barker 2014].

$$\mathrm{I}_{ij}^{SP} = \frac{\partial \mathrm{A}_{ij}^{SP}}{\partial \mathrm{A}_{ij}} = \prod_{k \neq i}^{n} \left[1 - \prod_{l=1}^{m} (1 - \mathrm{A}_{kl}) \right] \times \prod_{l \neq j}^{m} (1 - \mathrm{A}_{il}) \tag{37}$$

In examination of SP system it was noted that if a component I_{ij}^{SP} was ranked as the most important in terms of availability to the overall system, it would remain the dominant path and hence keep its designation as most important component even after substantial improvement! This is curious and leads to a possible conundrum when faced with resource constraints that only allow one component for redesign. It is therefore crucial to understand that the components in

the same sub group n as the most important component should also be considered for redesign. This is due to the parallel nature of the subsystem n and since it exists as a parallel subsystem from the original design, the requirement for overall system redundancy is continued to be assumed.

2.3.4. Parallel Series System

For a combination system, such as Parallel Series (PS), as shown in Figure 12. Parallel Series (PS) System, there are m sub components in parallel with n sub-systems in series. This type of overall system allows for built in redundancy at the m subsystem level, however, determination of the most important component also becomes somewhat complicated.

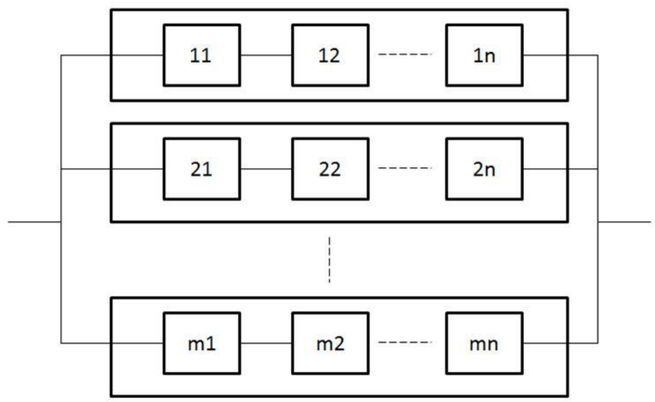

Figure 12. Parallel Series (PS) System

The derived availability importance of a Parallel Series (PS) system is as follows [Gravette and Barker 2014].

$$I_{ji}^{\mathrm{PS}} = \frac{\partial A^{\mathrm{PS}}}{\partial A_{ji}} = \prod_{l \neq i}^{m} \left(1 - \prod_{k=j}^{n} A_{lk} \right) \times \prod_{k \neq j}^{n} A_{jk} \qquad (38)$$

In examination of PS system it is noted the component I_{ij}^{PS} with the highest rank is the most important in terms of availability to the overall system. Redesign of this component would lead to the other components Availability Importance Measures (AIM) to intuitively increase overall and as expected.

2.4. Point Estimates and Random Variables

In the calculation for availability, a mean time is required for each element in the equation. The expected value, or mean, is a point estimate and are highly dependent upon the sample taken of the population. Additionally, the “mean” is a single point within the body of the sample and it does not describe anything about the variance or possible outliers, information needed to build a basic confidence interval.

Point estimates are ultimately a single observation within a sample that may or may not be the true expected value of the population. Two separate samples from the same population can have large differences between their expected values which when included together creates yet another but possibly more accurate overall expected value. Most statistics courses seek to prove that any sample from a population follows a normal distribution but that the underlying population can be represented by a non-normal distribution.

If instead of using a single point estimate in the availability calculation, a probability distribution function is used that represents both the uptime of the system or component as well as the downtime of the system or component. Perhaps such a setup is a more meaningful measure of determining importance and propagates more efficient use of valuable resources.

When deciding on what is most important in a system or component, understanding the body of observation, through the use and knowledge of a random variable, could possibly improve the precision of the decision framework. If a random variable such as a Weibull distribution is used to describe the uptime of a piece of equipment [Lie et al. 1977], therein exists a possible advantage over the information provided by just a single point estimate or expected value. Similarly, the same argument holds true for using a distribution to describe the downtime of a system. In Lie et al. [1977], it is discussed that the downtime or time to repair data follows a lognormal distribution.

It is with these ideas and understanding that this thesis seeks to improve the availability importance measure through the use the Monte Carlo Simulation Method modeling availability importance of a system or components via random variable probability distributions for the uptime and downtime inputs.

When using MCS it is important to generate a large number of iterations, preferably on the order of tens of thousands. The larger the number of iterations, the more precise the results become [Vose 2008]. The random number generator and time needed to perform calculations becomes the limiting factor for MCS, however, proper sectioning of the problem can usually supplant this issue [Vose 2008].

Once a MCS simulation is performed and a random distribution is created, a means of comparison of the final output is required. This is solved by using Copeland's method.

Copeland's method was proposed by A. H. Copeland in an unpublished seminar at the University of Michigan on applications of mathematics to the social sciences [Al-Sharrah 2010]. It concerns the study of ranking objects that have different comparative information such as databases, voting, chemical reactance etc. [Al-Sharrah 2010]. Attention is focused on the item with the highest rank order for the underlying interested characterizing indicator. It is similar to other methods such as average rank by Bruggeman et al. [2004]; however, it does not guarantee the final ranking will be different for all comparisons [Al-Sharrah 2010]. In Sharrah's paper, it was demonstrated that the simple Copeland method was consistent and comparable to the Hasse diagram but it has some disadvantages in that there is a loss of some information due to data aggregation; however, it has many advantages, namely:

- It is a rapid nonparametric ranking tool for identifying best or worst objects

- It is systematic and easily computerized even for very large data set sizes
- It is transparent, clear to the user, and flexible enough to be adapted for many purposes in terms of the number of objects or indicators
- It is based on scientifically justified framework
- Expert judgment can be used to add weights to the indicators, if needed
- It has proven to be stable to variations in the data
- It is readily comparable to the Hasse diagram method but has simpler mathematical requirements and allows for the possibility of providing relative or categorical ranking.

Simply, Copeland's Method is a non-parametric relative ranking approach that determines a preferred rank based on many pairwise comparisons. All criteria are weighted equally and no information about the preference is needed. The pairwise comparison is plotted as a cumulative density function (CDF) of interest and predetermined percentiles along the curves are selected for comparison. The mathematical representation of Copeland score for alternatives a and b are as follows:

$$\mathrm{S}_j(a,b) = \begin{cases} S_{j-1}\,(a,b) + 1 \;\; x_{aj} < \; x_{bj} \\ S_{j-1}\,(a,b) - 1 \;\; x_{aj} > \; x_{bj} \\ S_{j-1}\,(a,b) \;\; x_{aj} = \; x_{bj} \end{cases} \tag{39}$$

And calculate the Copeland Score for all alternatives

$$\mathrm{CS}(a) = \sum_{b \neq a} S_m(a,b) \tag{40}$$

The alternative with the highest CS provides the measure of interest.

2.5. Simulation Method Outline

Paul Sheehy and Eston Martz created detailed instructions on how to use MiniTab to create random distributions and perform Monte Carlo Analysis [Sheehy and Martz 2012].

The first step is to identify the Transfer Equation (TE) which is the quantitative model of what is being explored. In this case it is the Availability Importance Measure for a particular system design.

Second, for each variable in the transfer equation, define the type of random distribution as required. For the Availability Importance Measures, the uptime is known to follow a Weibull distribution and the downtime is known to follow a lognormal distribution. For each distribution, their parameters are pre-defined.

Third, MiniTab is tasked to create a very large random data set for each input of the transfer equation along the order of 100,000 observations. As such, each iteration represents a random value that could be observed over a long enough time period for that particular defined distribution.

Lastly, once the columns of 100,000 observations for each components distribution are created, the transfer equation can be applied to simulate the probable outcomes. Given the large number of samples, the overall results are likely to be a reliable indication of the transfer equation output over time.

Analyses of the results are done by Copeland's method by ranking the Availability Importance Measures data distributions pre-determined percentiles to determine if and where each importance value outranks each other. As discussed in the Copeland's Method section, the alternative with the highest Copeland Score provides the measure of interest. Then that AIM from each method (MCS and TPE) will be improved equally and Copeland's Method will again be used to do a pairwise comparison between the MCS chosen component system availability result and the TPE chosen component system availability result to determine if MCS is more efficient than TPE.

3. Illustrative Example

Independence is assumed for all components of a system or a system of systems in that a failure of a component or system does not have an impact on other components. The failure of a component can affect the overall system to be in a functioning or non-functioning state. It is assumed that the system or systems of components are in a steady state. For lognormal, threshold is greater than or equal to zero.

The uptime and downtime of the components are assumed to have the following random variables as referenced earlier by [Lie et al. 1977]. For uptime or Time Between Failure, a random variable with a Weibull probability density function (PDF) is assumed and the shape, θ and scale, β parameters are annotated:

$$f(t) = \frac{\beta}{\theta} * \left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} \tag{41}$$

For downtime or Time To Repair, a random variable with a Lognormal probability density function (PDF) is assumed with the scale σ, location m and threshold θ as parameters are annotated:

$$f(t) = \frac{e^{-((\ln\left(\frac{t-\theta}{m}\right))^{2}/(2 * \sigma^{2}))}}{(t-\theta) * \sigma\sqrt{2\pi}}, t > \theta; m, \sigma > 0 \tag{42}$$

MiniTab 16 is used in all Monte Carlo Simulations and calculations. MiniTab can create any defined distribution in a random form. For each component availability importance measure, 100,000 random numbers are generated that follow the predefined Weibull or Lognormal distribution as desired.

3.1. Application and Data Simulation

Given the uptime and downtime components of the AIM are represented as appropriate random variable distributions for each component in the system, and MCS can take the AIM transfer equation, a practical application and data simulation are performed through all steps using Copeland's method for final ranking of each component and method.

It is of interest to determine if there is improvement between MCS over the TPE method and success is determined if the AIM MCS produces a greater overall system availability improvement as measured by Copeland's method and the traditional point estimate.

Two system configurations, Parallel Series (PS) and Series Parallel (SP) systems are simulated and the results analyzed.

3.1.1. Parallel-Series System Simulation

Several examples exist on the industrial plant floor where machinery contains a redundant system of series components similar to Figure 13. Parallel-Series , such as a large critical hydraulic system or the intensifier section of a water jet cell.

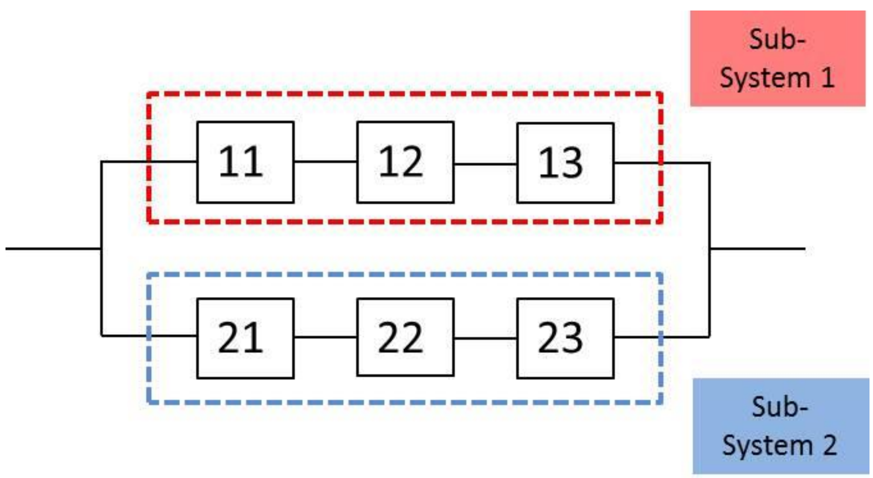

Figure 13. Parallel-Series System Example

In Figure 13. Parallel-Series System Example, there are two similar sub-systems in parallel that contain series components. The parallel redundancy of subsystem 1 and 2 allow for an increase in the possible chosen paths for the work to be performed. This increases the overall reliability and availability of the complete system if one of the components were to fail on the series section.

The availability importance equations will now be used in step one: *The appropriate transfer equation for the system layout*. For step two, *defining the input parameters of the transfer equation*, the uptime and downtime random variables are described for each type respectively as seen in Table 2. Initial Setup Parameters Parallel-Series System

Table 2. Initial Setup Parameters Parallel-Series System

Initial Setup Parameters	Weibull subsystem 1	Weibull subsystem 2
Parallel-Series, Uptime	**Shape/Scale**	**Shape/Scale**
Component 11, Component 21	0.9/5	10/4.25
Component 12, Component 22	12/4.75	10/4.25
Component 13, Component 23	10/5.25	10/5.25
Initial Setup Parameters	**Lognormal subsystem 1**	**Lognormal subsystem 2**
Parallel-Series, Downtime	**Location/Scale**	**Location/Scale**
Component 11, Component 21	1/0.25	1/0.25
Component 12, Component 22	1/0.5	1/0.5
Component 13, Component 23	1/0.8	1/0.8

Notice that there are different uptime parameters for some of the components due to the fact they may or may not be active in the system or they perform a duty and standby function where one subsystem has been operating a majority of the time. This reflects real world examples and will allow the AIM computation the ability to facilitate a definite choice for the most important component.

The Weibull distribution components that have a shape parameter of less than one exhibit an infant mortality failure rate. The shape parameters that are equal to one represent a random or constant failure rate. Those shape parameters that are greater than one represent end of life failure rates which are increasing with increasing time. This system is composed of mainly wear items that exhibit increasing failure rates except for one component that appears to be experiencing early life failures, component 11.

The Lognormal distribution components are described in terms of how MiniTab expects to define the random variable parameters such as the location m , scale σ and threshold θ. Overall, this system is assumed to be modular with repair or replacement components in inventory for quick turnaround during repair. This resembles several systems common in the industrial plant.

Step 3: *create random data* from the given random variable definitions and parameters in Table 2. Initial Setup Parameters Parallel-Series System with 100,000 iterations. Every component receives an entry column with both uptime and downtime random variable distributions created.

An example of the distributions created in Minitab is in Figure 14: Component 11 Weibull and Figure 15: Component 11 Lognormal:

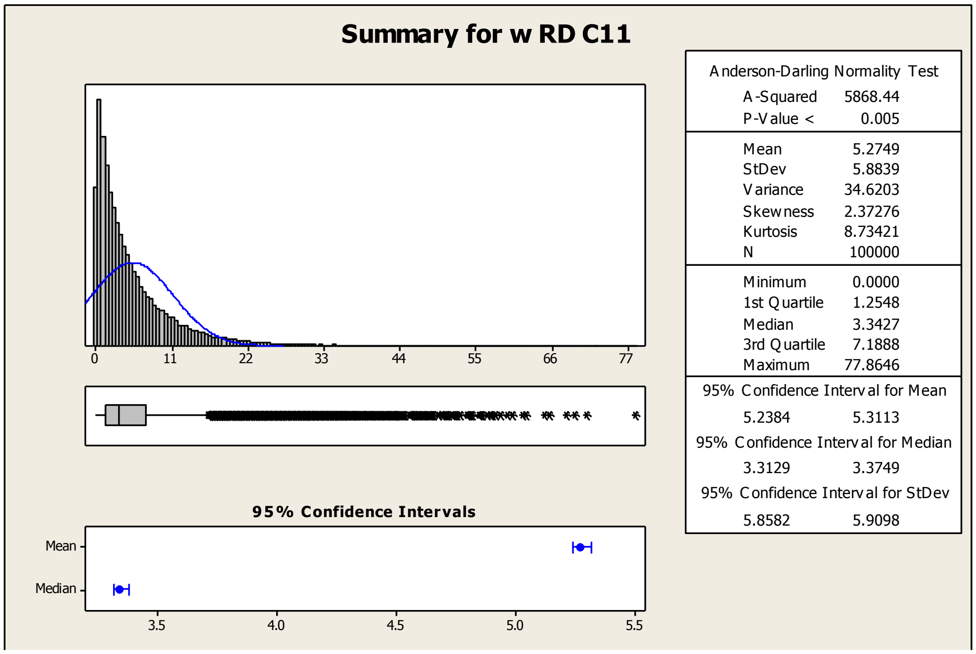

Figure 14: Component 11 Weibull

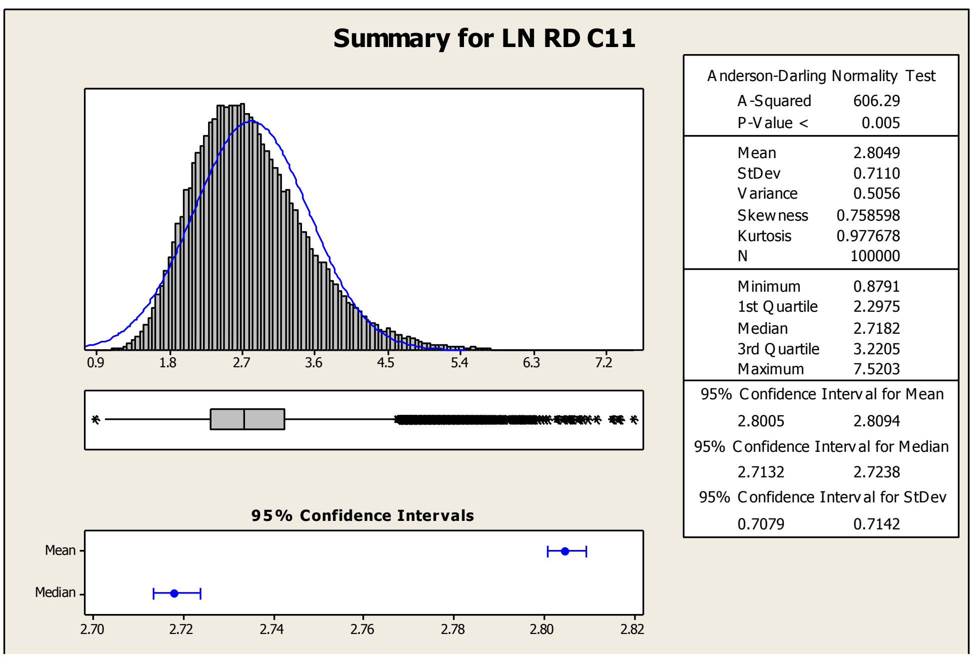

Figure 15: Component 11 Lognormal

Minitab includes helpful statistics about the summary distribution on the right side and below for reference.

Next, in step 4: *simulation* of each component is performed by breaking down the mathematical calculations into a few steps. After the uptime and downtime Weibull and Lognormal distributions are created, then each component's availability is calculated in an intermediate step

to result in a component level availability. An example of the component level availability is seen in Figure 16: Component 11 Availability.

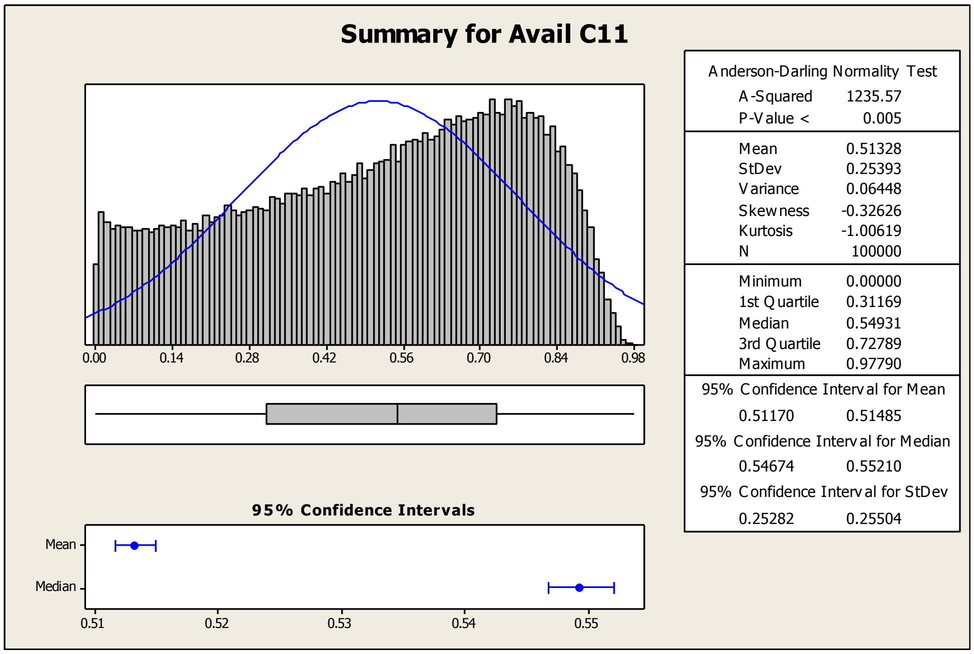

Figure 16: Component 11 Availability

This component level computed availability distribution is then pushed through the transfer equation to produce the final Monte Carlo Simulated Availability Importance Measure distribution. Each component's Availability Importance Measure distribution for the Parallel Series System is shown below.

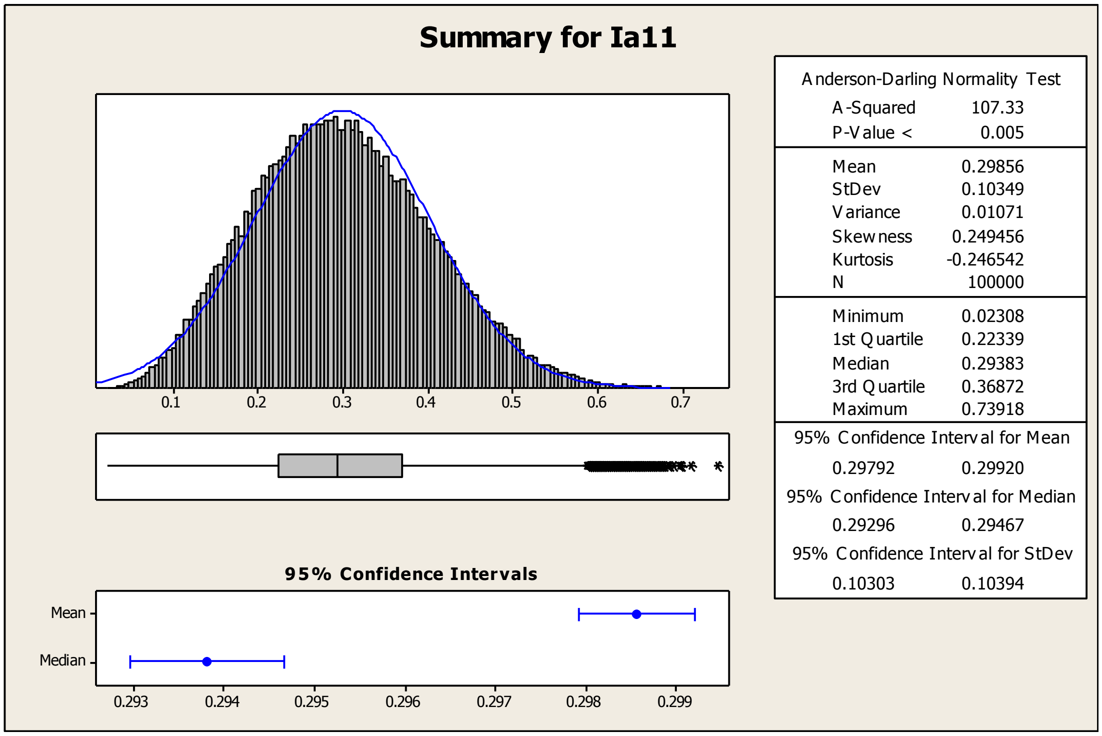

Figure 17: Component 11 AIM Distribution

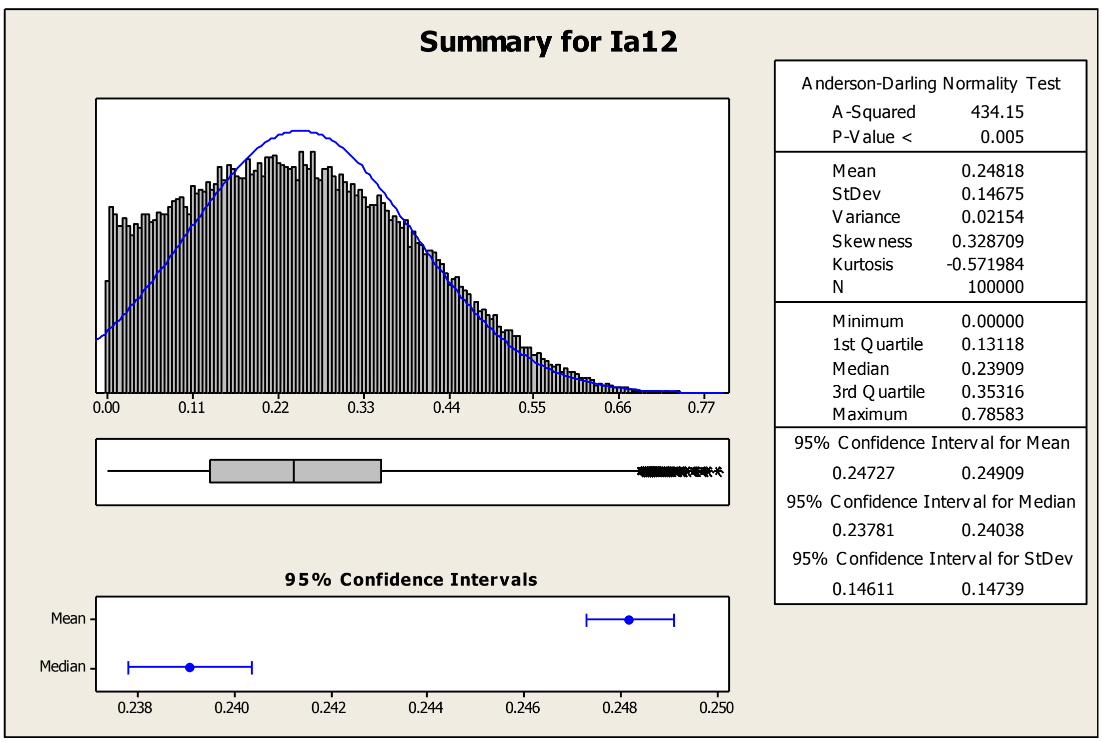

Figure 18: Component 12 AIM Distribution

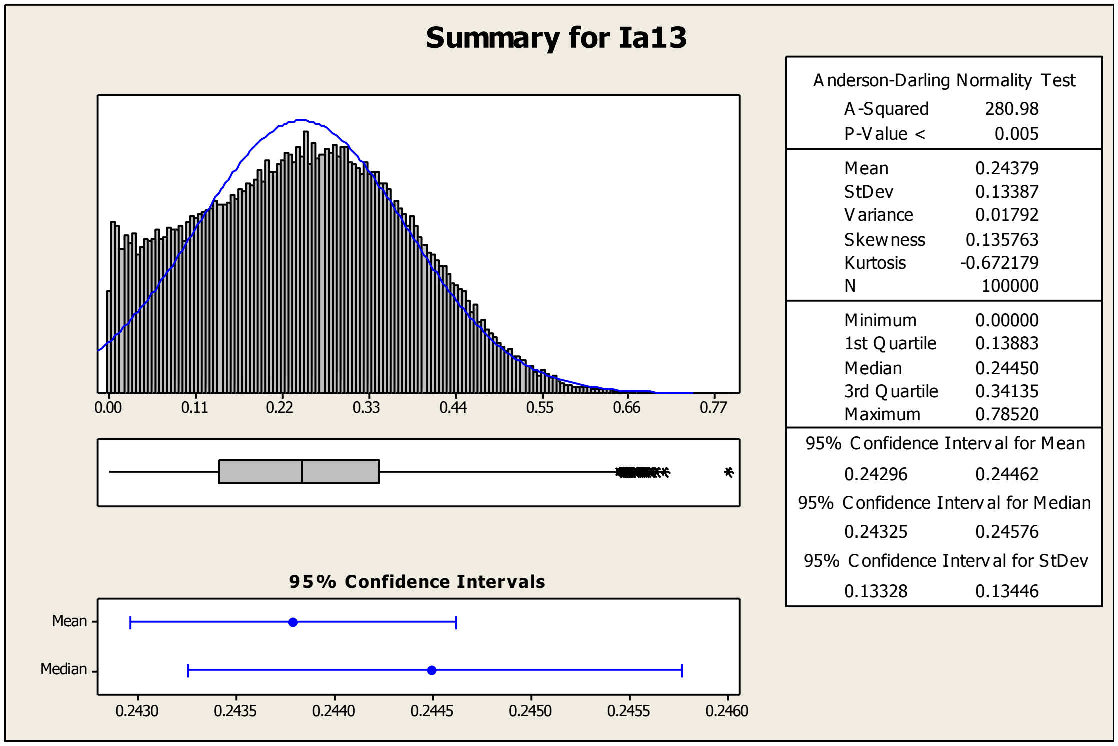

Figure 19: Component 13 AIM Distribution

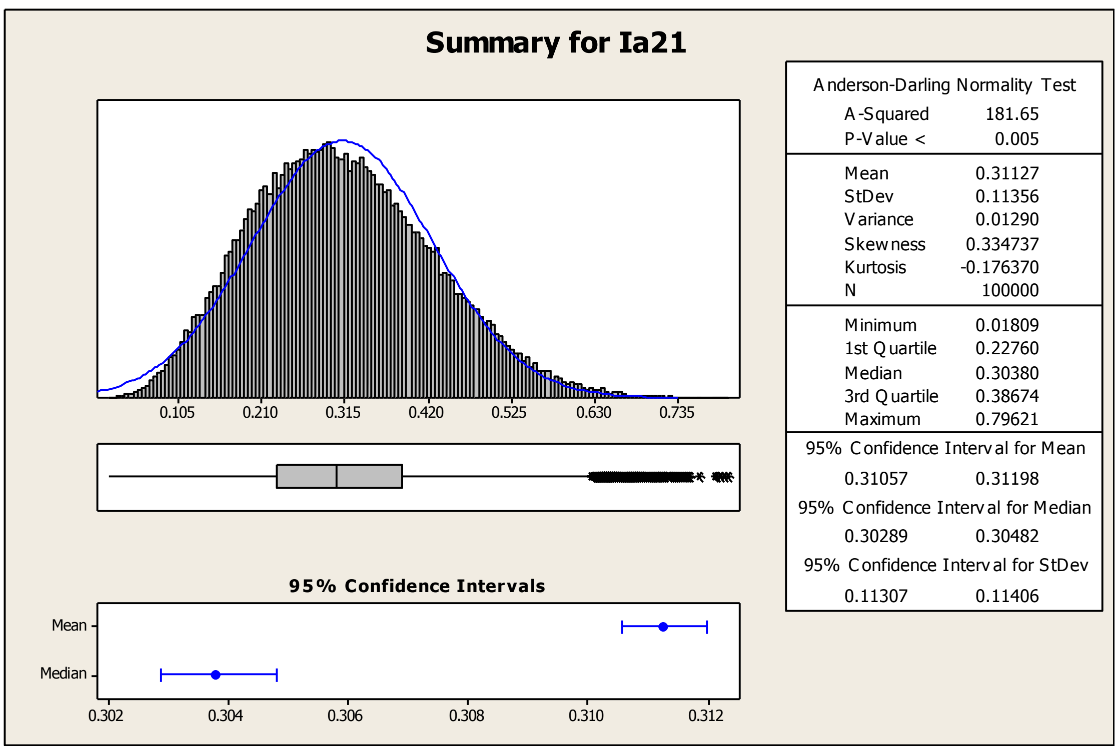

Figure 20: Component 21 AIM Distribution

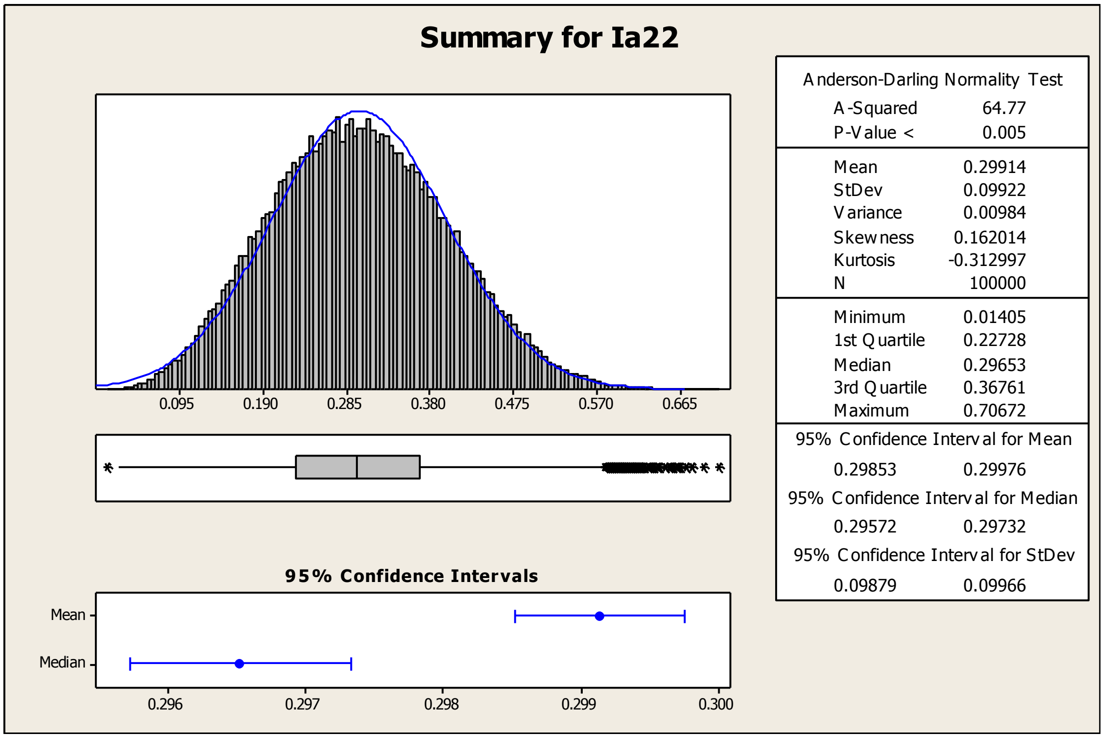

Figure 21: Component 22 AIM Distribution

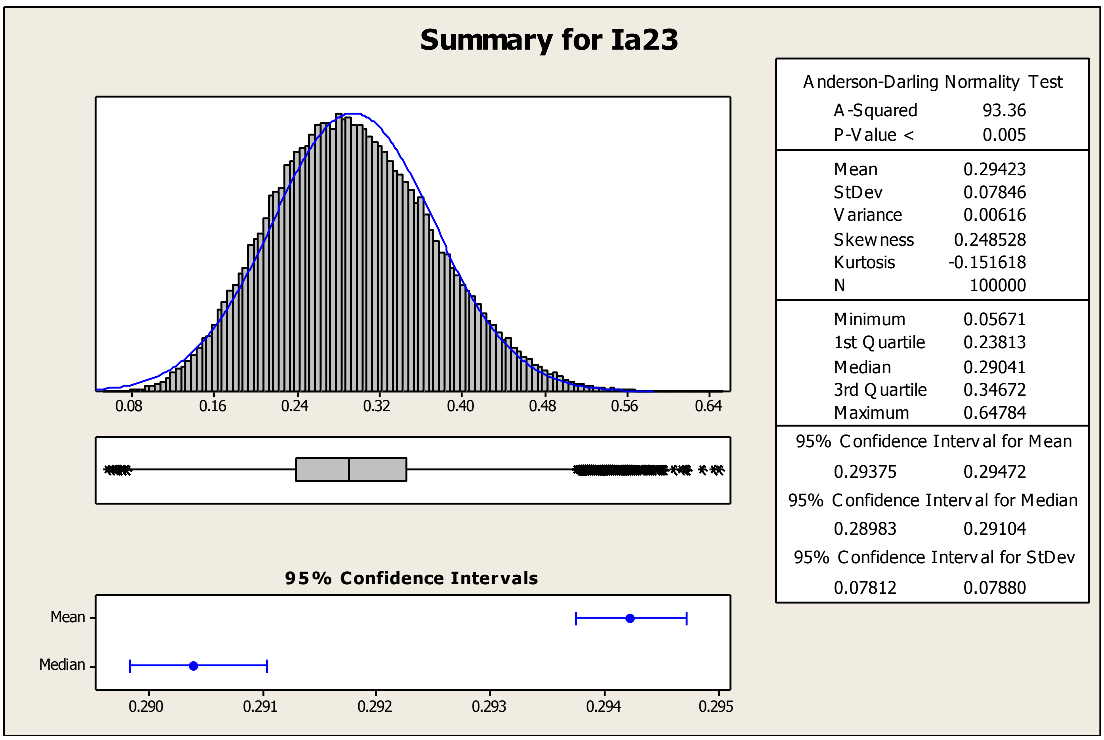

Figure 22: Component 23 AIM Distribution

Concurrent to the Monte Carlo Simulations, the same Availability Importance Measure (AIM) was performed for the Traditional Point Estimate (TPE) using only the traditional point estimates mean of each component distribution. For each distribution the expected value is one that would have traditionally been chosen for the "mean" in Mean Time Between Failure (MTBF) and the "mean" in Mean Time To Repair (MTTR) as seen below:

$$\text{MTBF} = \frac{\text{Uptime}}{\text{Count of system failures}} \tag{43}$$

$$\text{MTTR} = \frac{\text{Corrective Maintenance Downtime}}{\text{Count of system failures}} \tag{44}$$

Further clarification of only using the average of the distributions is that when we use the traditional point estimation method, we ignore all knowledge of the shape or characteristics of the distribution even though it exists for purposes of this thesis. The idea is that all we have is a count of system failures over a selected period of time giving a mean time or average time between failures. The same is assumed for the mean time to repair in the TPE method.

Table 3: Point Estimates for Parallel Series System shows the average of each random variable distributions that are then used to determine the Availability Importance Measures for the Traditional Point Estimation method.

Table 3: Point Estimates for Parallel Series System

Traditional Point Estimation	***Value***
Mean Weibull component 11	5.274881
Mean Lognormal component 11	2.804947
TPE Availability component 11	0.652846
Mean Weibull component 12	4.550231
Mean Lognormal component 12	3.080445
TPE Availability component 12	0.596308
Mean Weibull component 13	4.996404
Mean Lognormal component 13	3.737447
TPE Availability component 13	0.572073
Mean Weibull component 21	4.04472
Mean Lognormal component 21	2.806813
TPE Availability component 21	0.590338
Mean Weibull component 22	4.551903
Mean Lognormal component 22	3.069695
TPE Availability component 22	0.597237
Mean Weibull component 23	4.991835
Mean Lognormal component 23	3.735542
TPE Availability component 23	0.571974

Now that a baseline exists for the parallel series system described by Figure 13. Parallel-Series System Example and the Monte Carlo Simulation produced a final random variable output, Copeland's Method will now be used to compare the Availability Importance Measures from the MCS method to determine which component is the most important and a candidate for improvement. Recall that the component with the largest Copeland ranking will be considered the best candidate for improvement in the MCS method. Additionally, the component with the largest Availability Importance Measure from the Traditional Point Estimate (TPE) method will be considered for improvement to the system. Each component will receive the same magnitude improvement and a final Copeland Score against each method will determine which method is superior.

To perform the Copeland comparison, each AIM MCS distribution was reordered into a cumulative density function (CDF) and percentiles assigned from 1% to 100% in about ten percent intervals as seen in Table 4: Availability Importance Measures by Component for MCS PS example.

Table 4: Availability Importance Measures by Component for MCS PS example

%	#	Availability Importance Component 11	Availability Importance Component 12	Availability Importance Component 13
1%	1000	0.089293294	0.00465775	0.004768686
10%	10000	0.165920924	0.055227609	0.058463312
20%	20000	0.206978671	0.107604817	0.113873281
30%	30000	0.238679483	0.15381208	0.162538783
40%	40000	0.266830809	0.197142293	0.205232673
50%	50000	0.293826008	0.239090415	0.244498834
60%	60000	0.321456552	0.281967584	0.282218743
70%	70000	0.351576552	0.327835527	0.321034326
80%	80000	0.387353645	0.380387169	0.363994587
90%	90000	0.436556504	0.449565851	0.421642831
100%	100000	0.739178544	0.785826652	0.785202958

%	#	Availability Importance Component 21	Availability Importance Component 22	Availability Importance Component 23
1%	1000	0.088901697	0.091727823	0.131597871
10%	10000	0.168612379	0.170900003	0.194983896
20%	20000	0.210467646	0.211328903	0.225998712
30%	30000	0.244065967	0.242325683	0.249515284
40%	40000	0.274593813	0.270018343	0.270315687
50%	50000	0.303797292	0.296527342	0.290403815
60%	60000	0.334668229	0.323232147	0.311244441
70%	70000	0.36806281	0.351893439	0.334038799
80%	80000	0.408213404	0.38508705	0.360797069
90%	90000	0.464279568	0.430586239	0.398535658
100%	100000	0.796208996	0.706716115	0.647842651

Then Copeland Method of pairwise comparisons was performed and a ranking of components by largest Availability Importance is seen in Table 5: MCS Copeland Pairwise AIM Ranking PS Example.

Table 5: MCS Copeland Pairwise AIM Ranking PS Example

Monte Carlo Simulation Component	*Copeland Score*	*Rank*
Avail. Importance Component 11	29	4
Avail. Importance Component 12	11	6

Avail. Importance Component 13	12	5
Avail. Importance Component 21	47	1
Avail. Importance Component 22	36	2
Avail. Importance Component 23	30	3

Copeland's Method identified component 21 from the Monte Carlo Simulation method as the most important to the system in terms of affecting the overall availability. For the Traditional Point Estimation method, the following Availability Importance Rank was produced and shown in Table 6.

Table 6: TPE AIM Ranking PS Example

Traditional Point Estimation	***Value***	***Rank***
TPE Availability Importance Component 11	0.27234	4
TPE Availability Importance Component 12	0.29816	2
TPE Availability Importance Component 13	0.31079	1
TPE Availability Importance Component 21	0.26553	5
TPE Availability Importance Component 22	0.26246	6
TPE Availability Importance Component 23	0.27405	3

The Traditional Point Estimation method identified component 13 from the Traditional Point Estimation method as the most important to the system in terms of affecting the overall availability. The results are curious in that they are nearly on opposite ends of a ranking spectrum. The Monte Carlo Simulation resultant distribution places more emphasis on the second half of the system in contrast to the traditional point estimation that favors the first half of the system.

The research methodology has been formulated in this task. However, the journal manuscript and illustrative example still need much work.

TASK 3. IMPROVING INSPECTIONS AND COMPARING SUPPLIERS

This section is based on the following:
Hague, R.K., K. Barker, and J.E. Ramirez-Marquez. 2015. Interval-valued Availability Framework for Supplier Selection Based on Component Importance. Accepted in *International Journal of Production Research*.

1. Introduction

The Government Accountability Office [2011] recently found that the Department of Defense (DoD) does not effectively consider tradeoffs among cost, schedule, and performance when analyzing system requirements. The DoD has recently adopted a "Better Buying Power" mantra [Defense AT&L 2011], identifying 23 efficiency-related initiatives, including mandating affordability within system requirements. Ashton B. Carter [2010], Under Secretary of Defense, stated "…we have a continuing responsibility to procure the critical goods and services our forces need in the years ahead, but we will not have ever-increasing budgets to pay for them. We must therefore strive to achieve what economists call productivity growth: in simple terms, to do more without more." One particular area of need within DoD is in considering maintenance resources during supplier selection (e.g., for the F-35 Joint Strike Fighter [GAO 2013]). To reduce costs and answer Mr. Carter's statement, a proactive maintenance philosophy that the DoD has adopted is Reliability Centered Maintenance (RCM), which guides what must be done to ensure that a system continues to performs in its present operating context [Moubray 1997].

A primary goal of a maintenance organization is to minimize equipment downtime. In the DoD context, this translates to maximizing the availability of weapon systems. Availability, or the probability that a system is performing its required function at a given point in time when operated and maintained in a prescribed manner [Ebeling 2010], inherently accounts for reliability (the ability to last as long as intended) and maintainability (the ability to be fixed with minimum effort and time).

Routinely in the DoD, reliability and availability have been the focus of maintenance decision making for weapon systems, but not necessarily for the individual parts or components that make up the system. To build an available system, availability must be considered at the component level. In this paper, we focus on making appropriate supplier selection decisions to emphasize component availability. We assume that several suppliers can provide the same component part but with varying levels of reliability (mean time between failures) and maintainability (mean time to repair), the two constituents of availability. As such, we develop a supplier selection framework driven by component availability importance.

This paper addresses the need to make acquisition decisions with system availability in mind, proposing an availability-based sole supplier selection framework that accounts for uncertainty in the reliability and maintainability perspectives of availability. In doing so, we provide a supplier selection framework to "do more without more" by accounting for availability, thereby addressing concepts of reliability and maintainability in the procurement process. Often, supplier selection decisions ultimately come down to procurement cost, though we look beyond to

availability, a driver of subsequent costs. We extend a traditional weighted multi-criteria discrete comparison technique, TOPSIS, to make this comparison. The innovations in this work are (i) our treatment of the availability of each component as the criteria with which to select a supplier, and (ii) our use of component importance measures to derive how each component is weighted. Section 2 provides background to several methodologies integrated in the proposed framework. Section 3 develops the supplier selection framework with an illustration, and conclusions are given in Section 4.

2. Methodological Background

This section provides background on availability and importance measure calculations, interval arithmetic, and some approaches for making comparisons among discrete alternatives.

2.1. Availability

Mentioned previously, availability is a very common measure in reliability engineering, particularly for weapon systems whose function is needed at a moment's notice. *Availability* is calculated from *uptime* and *downtime*. Uptime, a function of reliability, is defined as the average time during which an asset or system is either fully operational or is ready to perform its intended function. Downtime measures how long a system is not in function, likely due to maintenance activities, suggesting that downtime is synonymous with maintainability. The traditional functional relationship for availability is shown in Eq. (45), with mean time between failure (MTBF) as a measure of uptime and mean time to repair (MTTR) as a measure of downtime [Lie et al. 1977].

$$\text{Availability} = \frac{\text{uptime}}{\text{uptime} + \text{downtime}} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{45}$$

Availability is a key metric that a maintenance organization (or any other) can use to measure its effectiveness. If a system or component is operating for a longer period between failures and has a minimal corrective maintenance time, then its availability will likely be at a desirable level.

2.2. System Configurations and Importance Measures

A system is comprised of multiple components or subsystems. Common system configurations are shown in Figure 1. A simple series system with *n* components, where each component must be in operating condition for the system to operate, is represented in Figure 1a. Figure 1b portrays a parallel system with *m* components, where each individual component does not have to be in operating condition for the system to operate due to redundancy. Slightly more complex system structures appear in Figure 1c, a series-parallel system (a series of *n* subsystems each with parallel component configurations). Yang et al. [2011] provide an example series-parallel system within an aircraft: the servo-actuation system, which consists of servo controllers, servo actuators, power sources, and an actuating cylinder. Figure 1d, a parallel-series system (*m* series subsystems in parallel). Gravette and Barker [2014] provide a realistic parallel-series system example that would appear in a DoD weapons system.

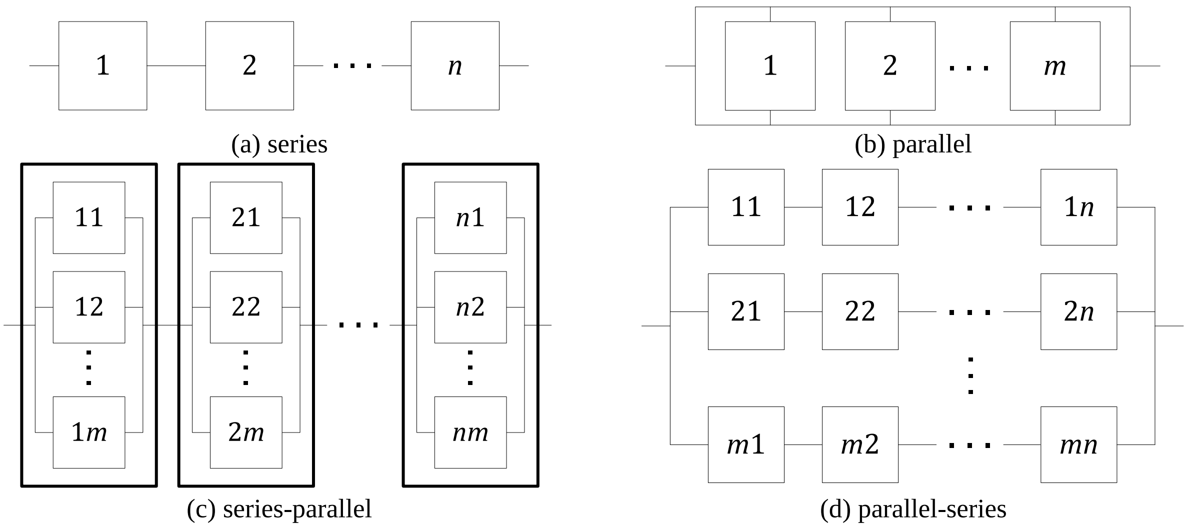

Figure 23. Four primary configurations that describe the structure of most systems.

The illustrative example used subsequently in this paper is the aircraft servo-actuation series-parallel system. In a series-parallel system, there are multiple components, each with their own criticality to the performance of the system. We can measure the importance of each component in contributing to overall system performance with the calculation of component importance measures (CIMs). CIMs allow for the ranking of components from most important to least.

Reliability is the most common measure of system performance for applying CIMs [Kuo and Zuo 2003, Modarres et al. 2010]. That is, in the reliability context, CIMs highlight the components that are most critical to system reliability. CIM examples include risk reduction worth (RRW), an index that quantifies the potential damage to a system caused by a particular component, and the reliability achievement worth (RAW) of a component, or the maximum proportion increase in system reliability generated by that component [Ramirez-Marquez et al. 2006]. This work will focus on the Birnbaum CIM [Birnbaum 1969], shown in Eq. (46). Where R_s measures system reliability and R_i measures the reliability of component *i*, the Birnbaum CIM, I_i^B, measures the change in system reliability due to a change in the reliability of component *i*. The component with the largest I_i^B value is the component that offers the greatest improvement in system reliability when its reliability is improved.

$$\mathrm{I}_i^B = \frac{\partial R_s}{\partial R_i} \tag{46}$$

As availability is the primary system performance measure of interest in this paper, we adopt a Birnbaum importance measure for availability [Cassady et al. 2004, Barabady and Kumar 2007,

Gravette and Barker 2014], shown in Eq. (47). The availability of the system and the availability of component *i* are represented with A_s and A_i, respectively.

$$I_i^A = \frac{\partial A_s}{\partial A_i} \tag{47}$$

The availability of a series-parallel system is provided in Eq. (8). Applying Eq. (47) to this series-parallel system results in the importance measure in Eq. (49) [Gravette and Barker 2014].

$$A^{\mathrm{SP}} = \prod_{i=1}^{n}\left[\coprod_{j=1}^{m} A_{\mathrm{a}_{ij}}\right] = \prod_{i=1}^{n}\left[1-\prod_{j=1}^{m}\left(1-\frac{\mathrm{MTBF}_{ij}}{\mathrm{MTBF}_{ij}+\mathrm{MTTR}_{ij}}\right)\right] \tag{48}$$

$$\begin{aligned}\mathrm{I}_{ij}^{\mathrm{SP}} = \frac{\partial \mathrm{A}^{SP}}{\partial \mathrm{A}_{ij}} &= \prod_{k\neq i}^{n}\left[1-\prod_{l=1}^{m}(1-\mathrm{A}_{kl})\right]\times\prod_{l\neq j}^{m}(1-\mathrm{A}_{il}) \\ &= \prod_{k\neq i}^{n}\left[1-\prod_{l=1}^{m}\left(1-\frac{\mathrm{MTBF}_{kl}}{\mathrm{MTBF}_{kl}+\mathrm{MTTR}_{kl}}\right)\right]\times\prod_{l\neq j}^{m}\left(1-\frac{\mathrm{MTBF}_{il}}{\mathrm{MTBF}_{il}+\mathrm{MTTR}_{il}}\right)\end{aligned} \tag{49}$$

We will subsequently use Eq. (49) to rank the importance of system components to prioritize suppliers of these components.

2.3. Comparing Discrete Alternatives

Often decision makers prefer not necessarily to choose the best option but avoid the worst option. The Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS) [Hwang and Yoon 1981] is a tool that addresses this decision making preference with the concept of the compromise solution, or the option that is nearest to the best solution (or positive ideal solution) and farthest from the worst solution (or negative ideal solution). The idea behind TOPSIS is rooted in reference-dependent theory, wherein consumers evaluate alternatives in terms of gains and losses to a subjective reference point [Kahneman and Tversky 1979]. Multiple criteria are considered when determining the positive and negative ideal solution, and those criteria are weighted separately depending on decision maker preferences. Several recent applications of comparing discrete alternatives with TOPSIS include project selection [Khalili-Damghani et al. 2013, Taylan et al. 2014], manufacturing decision making [Azadeh et al. 2011, Goyal et al. 2012], and enterprise systems [Rouhani et al. 2012, Ye 2010].

For B different discrete alternatives, $b = 1, \dots, B$, and Cdifferent objectives or performance criteria, $c = 1, \dots, C$, each alternative exhibits performance ratings contained in set $X = \{x_{bc} | b = 1, \dots, B; c = 1, \dots, C\}$. As some criteria may be more important than others to the decision maker, the criteria are weighted with w_c, $c = 1, \dots, C$. Performance ratings x_{bc} can be normalized if the various performance criteria exhibit different ranges (e.g., reliability on [0,1] along with costs in millions of dollars) with a variety of normalization approaches. For the application discussed subsequently, normalization will not be necessary.

The weighted performance rating of alternative b for criterion c is found in Eq. (50). A number of approaches to assess attribute weights from decision makers could be used, including the Analytical Hierarchy Process [Saaty 1990] or rank reciprocal approach [Barron and Barrett 1996]. We will describe the use of the availability CIM for determining this weight in Section 3.

$$v_{bc} = w_c x_{bc} \tag{50}$$

The positive ideal solution has all the best attainable criteria values, while the negative ideal solution has all worst possible criteria values. The positive ideal solution, B^+, is found with Eq. (51). Set C^+ represents the set of benefit criteria, where larger values of the criteria are preferred (e.g., profit, time between failure). Set C^- is the set of cost criteria, where smaller values of the criteria are preferred (e.g., expenditures, losses, travel time). Eq. (51) suggests that the positive ideal solution consists of those weighted performance ratings that maximize benefit criteria and minimize cost criteria. Likewise, the negative ideal solution, or the weighted performance ratings that represent the smallest from set C^+ and largest from set C^-, is provided in Eq. (52).

$$B^+ = \{v_1^+, \dots, v_c^+, \dots, v_C^+\} = \left\{\left(\max_b v_{bc} \,|c \in C^+\right), \left(\min_b v_{bc} \,|c \in C^-\right)\right\} \tag{51}$$

$$B^- = \{v_1^-, \dots, v_c^-, \dots, v_C^-\} = \left\{\left(\min_b v_{bc}|c \in C^+\right), \left(\max_b v_{bc}|c \in C^-\right)\right\} \tag{52}$$

The Euclidean distance between the performance ratings of alternative b and B^+ is found in Eq. (53). The distance from the positive ideal solution for alternative b is referred to as D_b^+. Likewise, the Euclidean distance between alternative b and B^- is found in Eq. (54) and is referred to as D_b^-.

$$D_b^+ = \sqrt{\sum_{c=1}^{C} (v_{bc} - v_c^+)^2} \tag{53}$$

$$D_b^- = \sqrt{\sum_{c=1}^{C} (v_{bc} - v_c^-)^2} \tag{54}$$

The preference order of alternatives can then be generated by ordering the measure in Eq. (55) in descending order. $D_b^\star$ is a measure of the similarity to the positive ideal solution.

$$D_b^\star = \frac{D_b^-}{D_b^+ + D_b^-} \tag{55}$$

2.4. Interval Arithmetic

Point estimates (e.g., MTBF, MTTR) often do not effectively portray the uncertainty associated with their underlying random variables (e.g., time between failures, repair time). In this work, we opt to not use point estimates for failure time and repair time. An approach where these uncertain

parameters are described by probability distributions is always preferred when distributions are known, as one could address the problem with, for example, Monte Carlo simulation. However, when such probability distributions are not known, "forcing" a distribution may do more harm to the decision making process than good [Huber 2010]. This is particularly true when developing distributions for failure time or repair time during the requirements development process in system design.

Addressing such uncertainty in the TOPSIS technique has been done with an extension using fuzzy numbers to deal with uncertainty in the set of performance ratings, X [e.g., Samvedi et al. 2013, Vahdani and Zandieh 2010, Chen et al. 2006]. We instead represent uncertainty in these failure time and repair time parameters with interval values, assuming we can bound the parameters with minimum and maximum values. If we can only assume the upper and lower bounds, we should "consider what decisions we could reach for all possible values of those data that are consistent with those interval constraints" [Huber 2010].

An interval number is an ordered pair of real numbers $\left[\underline{y}, \overline{y}\right]$ such that $\underline{y} \leq \overline{y}$, where the underbar represents the lower bound of the interval and the overbar represents the upper bound. For interval numbers $Y = \left[\underline{y}, \overline{y}\right]$ and $Z = [\underline{z}, \overline{z}]$, the following algebraic relationships hold [Moore 1966].

$$Y + Z = \left[\underline{y}, \overline{y}\right] + [\underline{z}, \overline{z}] = \left[\underline{y} + \underline{z}, \overline{y} + \overline{z}\right] \tag{56}$$

$$Y - Z = \left[\underline{y}, \overline{y}\right] - [\underline{z}, \overline{z}] = \left[\underline{y} - \overline{z}, \overline{y} - \underline{z}\right] \tag{57}$$

$$\begin{aligned} Y \times Z &= \left[\underline{y}, \overline{y}\right] \times [\underline{z}, \overline{z}] \\ &= \left[\min\left(\underline{y} \times \underline{z}, \underline{y} \times \overline{z}, \overline{y} \times \underline{z}, \overline{y} \times \overline{z}\right), \max\left(\underline{y} \times \underline{z}, \underline{y} \times \overline{z}, \overline{y} \times \underline{z}, \overline{y} \times \overline{z}\right)\right] \end{aligned} \tag{58}$$

$$Y/Z = \left[\underline{y}, \overline{y}\right] / [\underline{z}, \overline{z}] = \left[\min\left(\underline{y}/\underline{z}, \underline{y}/\overline{z}, \overline{y}/\underline{z}, \overline{y}/\overline{z}\right), \max\left(\underline{y}/\underline{z}, \underline{y}/\overline{z}, \overline{y}/\underline{z}, \overline{y}/\overline{z}\right)\right], \text{ where } 0 \notin [\underline{z}, \overline{z}] \tag{59}$$

$$Y^2 = \left[\min\left(\underline{y}^2, \left|\underline{y} \times \overline{y}\right|, \overline{y}^2\right), \max\left(\underline{y}^2, \left|\underline{y} \times \overline{y}\right|, \overline{y}^2\right)\right] \tag{60}$$

Other properties include the following [Neumaier 1990].

$$1/Y = [1/\overline{y}, 1/\underline{y}]\text{, where } 0 < y_1 < y_2 \tag{61}$$

$$\alpha \times Y = \alpha \times \left[\underline{y}, \overline{y}\right] = \left[\underline{y}, \overline{y}\right] \times \alpha = \left[\alpha \times \underline{y}, \alpha \times \overline{y}\right]\text{, for real constant } \alpha \geq 0 \tag{62}$$

Ultimately, there are instances where two intervals will be compared to each other (e.g., determining which suppliers' interval availability is preferred to another). For intervals $Y = \left[\underline{y}, \overline{y}\right]$ and $Z = [\underline{z}, \overline{z}]$, assume that Y is preferred to Z when a maximum value of the interval is sought. Barker and Rocco [2011] provide several decision rules for comparing intervals shown in Eq. (63) that reflect different levels of risk aversion.

$$Y \succ Z \iff \begin{cases} \underline{y} > \underline{z} & \text{Best case} \\ \overline{y} > \overline{z} & \text{Worst case} \\ \left(\underline{y} + \overline{y}\right) > (\underline{z} + \overline{z}) & \text{Laplace} \\ \theta\left(\underline{y} - \underline{z}\right) > (1-\theta)(\overline{y} - \overline{z}), \theta \in [0,1] & \text{Hurwicz} \\ (\overline{y} - \underline{z}) > \left(\overline{z} - \underline{y}\right) & \text{Min regret} \end{cases} \tag{63}$$

3. Framework for Supplier Selection with Illustrative Example

Dickson [1966] introduced 23 supplier selection criteria still found in literature today, including quality, delivery, performance history, and price. Many have recently applied TOPSIS to a subset of these criteria for supplier selection [Kasirian and Yusuff 2013, Liao and Kao 2011, Awasthi et al. 2010, Wang et al. 2009]. In this work, we focus on the *availability* aspect of supplier quality. That is, we want to select component (or service) suppliers based on their ability to maintain a level of availability in the system of interest. And an innovation of this work comes from how we weight the importance of component availability with the availability CIM provided in Eq. (47). Ultimately, we choose a sole supplier who can supply the most important components of the system such that system availability is maintained.

We assume that the desired component and system availability can be derived from system requirements or from documentation from the original equipment manufacturer (OEM). We will derive component importance from these original requirements and later compare how different suppliers meet these availability requirements. However, when we assume that component design specifications come from system requirements, we could naturally conclude some uncertainty associated with the two main elements of the availability calculation, MTBF and MTTR. Such uncertainty could exist particularly when a new system is under development or is being redesigned, and failure or repair histories do not exist. Assume, however, that intervals can effectively quantify these design parameters. Notation for the intervals of the two availability parameters are $\text{MTBF} = \left[\underline{\text{MTBF}}, \overline{\text{MTBF}}\right]$ and $\text{MTTR} = \left[\underline{\text{MTTR}}, \overline{\text{MTTR}}\right]$.

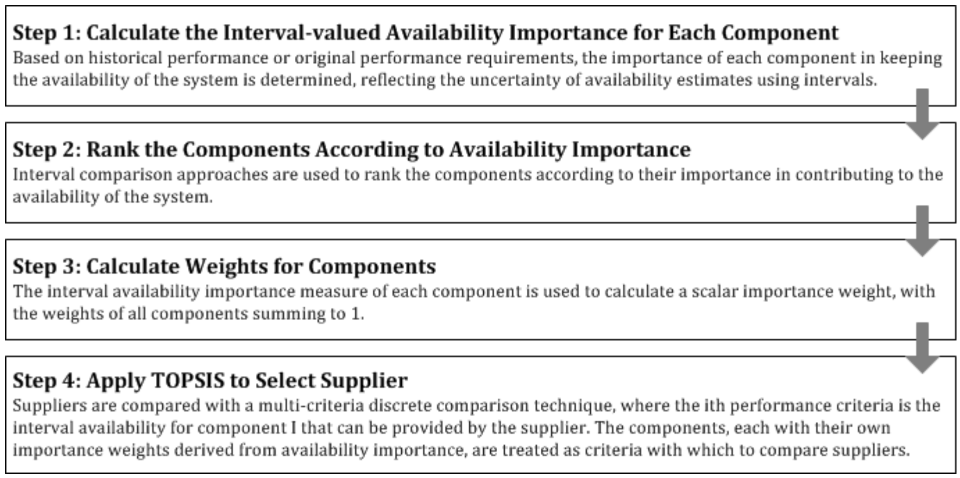

Figure 24. Overview of the four-step framework for availability-based supplier selection.

The following subsections develop the four steps of the interval-valued availability framework for supplier selection, the outline for which is provided in Figure 24. The series-parallel configuration in Figure 25 will illustrate framework within each step. The system, which mentioned previously could represent an aircraft servo-actuation system, consists of three subsystems in series, where each subsystem is a collection of components (servo controllers, servo actuators, and power sources) arranged in parallel. While this example is notional, a servo-actuation system is an important subsystem in an aircraft flight control system. Other configurations beyond the series-parallel system could be explored, including network configurations, assuming that computational requirements for calculating system availability and availability component importance are not too great.

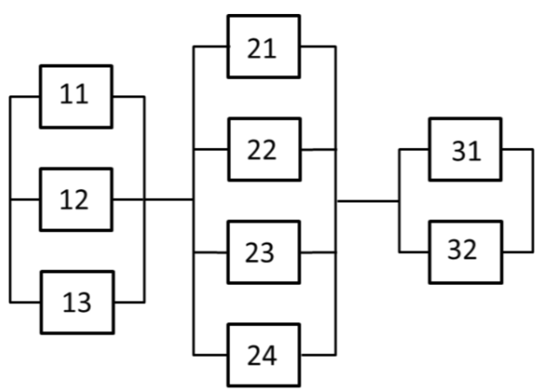

Figure 25. The series-parallel system serving as the illustrative example for the supplier selection framework.

3.1. Step 1: Calculate the Interval-valued Availability Importance for Each Component

Before considering any of the suppliers, we want to understand the importance of each component with respect to its contribution to system availability. For the general series-parallel representation in Figure 1c, Eq. (64) integrates the availability CIM with the interval representations of mean failure and repair times.

$$
\mathrm{I}_{ij}^{SP} = \frac{\partial \mathrm{A}^{SP}}{\partial \mathrm{A}_{ij}} = \prod_{k \neq i}^{n} \left[1 - \prod_{l=1}^{m} \left(1 - \frac{\left[\underline{MTBF}, \overline{MTBF}\right]_{kl}}{\left[\underline{\mathrm{MTBF}}, \overline{MTBF}\right]_{kl} + \left[\underline{\mathrm{MTTR}}, \overline{MTTR}\right]_{kl}} \right) \right] \times \prod_{l \neq j}^{m} \left(1 - \frac{\left[\underline{MTBF}, \overline{MTBF}\right]_{il}}{\left[\underline{MTBF}, \overline{MTBF}\right]_{il} + \left[\underline{\mathrm{MTTR}}, \overline{MTTR}\right]_{il}} \right) \tag{64}
$$

Applying the interval arithmetic rules in Eqs. (56), (59), and (62), the ratio in Eq. (64) becomes the following.

$$
\begin{aligned}
&\frac{\left[\underline{\mathrm{MTBF}}, \overline{\mathrm{MTBF}}\right]}{\left[\underline{\mathrm{MTBF}}, \overline{\mathrm{MTBF}}\right] + \left[\underline{\mathrm{MTTR}}, \overline{\mathrm{MTTR}}\right]} \\
&\quad = \Big[\min\Big\{\underline{(\mathrm{MTBF}}^2 + \underline{(\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), ((\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTBF}}) + (\underline{\mathrm{MTBF}} \\
&\quad \times \overline{\mathrm{MTTR}})), ((\overline{\mathrm{MTBF}} * \underline{\mathrm{MTBF}}) + (\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), (\overline{\mathrm{MTBF}}^2 + (\overline{\mathrm{MTBF}} \\
&\quad \times \overline{\mathrm{MTTR}})\Big\}, \max\Big\{\underline{(\mathrm{MTBF}}^2 + \underline{(\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), ((\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTBF}}) \\
&\quad + (\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})), ((\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTBF}}) + (\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), (\overline{\mathrm{MTBF}}^2 \\
&\quad + (\overline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})\Big\}\Big]
\end{aligned} \tag{65}
$$

Using constant ψ, the interval-valued availability importance measure is simplified in Eq. (67).

$$
\begin{aligned}
\psi = \Big[&\underline{(\mathrm{MTBF}}^2 + \underline{(\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), ((\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTBF}}) + (\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})), ((\overline{\mathrm{MTBF}} \\
&\times \underline{\mathrm{MTBF}}) + (\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), (\overline{\mathrm{MTBF}}^2 + (\overline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})\Big]
\end{aligned} \tag{66}
$$

$$
\mathrm{I}_{ij}^{SP} = \frac{\partial \mathrm{A}^{SP}}{\partial \mathrm{A}_{ij}} = \prod_{k \neq i}^{n} \left[1 - \prod_{l=1}^{m} (1 - [\min(\psi_{kl}), \max(\psi_{kl})]) \right] \times \prod_{l \neq j}^{m} (1 - [\min(\psi_{il}), \max(\psi_{il})]) \tag{67}
$$

The importance of the components to system availability is a function of the design of the system not the suppliers. Therefore, system requirements for MTBF and MTTR are used to parameterize Eq. (67). For the illustrative example in Figure 25, the interval bounds for the availability parameters from the system design requirements for each component are found in Table 7.

Table 7. Component MTBF and MTTR intervals, in days.

Component	$\underline{MTBF}$	$\overline{MTBF}$	$\underline{\mathrm{MTTR}}$	$\overline{MTTR}$
C_{11}	25	35	1	5
C_{12}	365	395	2	7
C_{13}	150	165	1	8

C_{21}	150	200	2	5
C_{22}	75	110	1	6
C_{23}	185	200	3	5
C_{24}	120	125	1	3
C_{31}	365	465	1	1.5
C_{32}	365	485	1	2

When applying the interval division rule in Eq. (59), there will always be an instance where the denominator is less than the numerator when dividing one interval's maximum by another's minimum. This is problematic when calculating availability values, as the definition of availability requires that it be on [0,1]. As such, we eliminate these possibilities with Eq. (68), a reformulation of the interval division arithmetic. Resulting component availability and importance measure calculations are found in Table 8. CIM results are given in several decimal places as some are very small in magnitude.

$$Y/Z = \left[\underline{y}, \overline{y}\right] / \left[\underline{z}, \overline{z}\right] = \left[\min\left(\underline{y}/\underline{z}, \underline{y}/\overline{z}, \overline{y}/\underline{z}, \overline{y}/\overline{z}\right), \max\left(\underline{y}/\underline{z}, \underline{y}/\overline{z}, \overline{y}/\underline{z}, \overline{y}/\overline{z}\right)\right], \text{ where } 0 \leq Y/Z \leq 1 \quad (68)$$

Table 8. Component availability intervals.

Component	$\underline{\mathrm{I}_{ij}^{SP}}$	$\overline{\mathrm{I}_{ij}^{SP}}$
C_{11}	0.000036089	0.012236505
C_{12}	0.000254704	0.049855491
C_{13}	0.000209593	0.034514925
C_{21}	0.000001735	0.002155172
C_{22}	0.000001735	0.001635931
C_{23}	0.000001430	0.005926724
C_{24}	0.000002762	0.009251472
C_{31}	0.002732233	0.250513347
C_{32}	0.002732233	0.217577706

3.2. Step 2: Rank the Components According to Availability Importance

As Eq. (67) is a function of interval values, the resulting availability CIM takes the form of an interval, as shown in Table 8. The larger the value of I_{ij}^{SP}, the more important is component *ij*. Discussed previously, a ranking of I_{ij}^{SP} provides a prioritization of components from most important to system availability to least. However, given that the availability CIM is interval-valued, ordering the components in Table 8 is not a straightforward task. For example, the intervals for components 11 and 12 overlap with each other, making them indistinguishable without a decision rule.

We use the Laplace criterion from Eq. (63) to show the order relationship when a maximum value is sought. The ranking of components appears in Table 9. Based on the system requirements, Table 9 suggests that the components 31 and 32, the components in subsystem 3,

are the most important in their contribution to system availability. Components within subsystem 2 would appear to be the least important. Different decision rules take different optimistic and pessimistic perspectives on the rankings of the intervals, though the Laplace rule is fairly risk neutral. The min regret rule from Eq. (63) also produces the same ranking.

Table 9. Ranking of components according to their interval-valued importance measure results.

Component	Laplace criterion $\left(\underline{\mathrm{I}_{ij}^{SP}} + \overline{\mathrm{I}_{ij}^{SP}}\right)$	Rank
C_{11}	0.0123	5
C_{12}	0.0501	3
C_{13}	0.0347	4
C_{21}	0.0022	8
C_{22}	0.0016	9
C_{23}	0.0059	6
C_{24}	0.0093	7
C_{31}	0.2532	1
C_{32}	0.2203	2

3.3. Step 3: Calculate Weights for Components

Mentioned previously, we adopt the TOPSIS approach for selecting among alternatives when different evaluation criteria are considered. In this application, we select a sole supplier for all components based on its ability to supply component parts with good availability. Therefore, we consider every component to be an "evaluation criterion" in the selection of a supplier.

The TOPSIS approach requires that a weight be applied to each evaluation criterion. The availability importance measure from Step 2 gives us a means to weight each component. To scale the Laplace criterion result from Table 9 such that all weights sum to 1, Eq. (69) is applied.

$$\mathrm{w}_{ij} = \frac{\left(\underline{\mathrm{I}_{ij}^{SP}} + \overline{\mathrm{I}_{ij}^{SP}}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\underline{\mathrm{I}_{ij}^{SP}} + \overline{\mathrm{I}_{ij}^{SP}}\right)} \tag{69}$$

The results of Table 9 and Eq. (69) provide an objective approach to weighting the components according to their availability importance based on system design requirements. The result is provided in Table 10.

Table 10. Component weighting using scaled interval-valued importance results.

Component	$\left(\underline{\mathrm{I}_{ij}^{SP}} + \overline{\mathrm{I}_{ij}^{SP}}\right)$	Weight
C_{11}	0.0123	0.0208

C_{12}	0.0501	0.0850
C_{13}	0.0347	0.0589
C_{21}	0.0022	0.0037
C_{22}	0.0016	0.0028
C_{23}	0.0059	0.0101
C_{24}	0.0093	0.0157
C_{31}	0.2532	0.4295
C_{32}	0.2203	0.3736

3.4. Step 4: Apply TOPSIS to Select Supplier

The final step in the framework is to select a sole supplier. As we are selecting a sole supplier, any supplier alternatives that are unable to meet the system requirements are not be considered: we only choose among those suppliers whose availability (via MTBF and MTTR) outperforms the requirements [Blanchard and Fabrycky 2010]. For each supplier S_b, we evaluate their availability for each component, criterion c. Availability is an interval number, $\left[\underline{A_{S_b,c}}, \overline{A_{S_b,c}}\right]$. The results from the TOPSIS analysis will provide the supplier that is closest to the best availability for each component and farthest from the worst availability for each component, the differences for which are weighted according to each component's importance and summed across all components.

Table 11 depicts four suppliers and their interval availability for each component. There is considerable overlap among the component availabilities for each supplier, therefore requiring an analytical approach to determine which supplier is ideal.

Table 11. Interval-valued component availabilities for each supplier.

	Supplier							
	S_1		S_2		S_3		S_4	
Component	$\underline{A_{S_1,c}}$	$\overline{A_{S_1c}}$	$\underline{A_{S_2c}}$	$\overline{A_{S_2c}}$	$\underline{A_{S_3,c}}$	$\overline{A_{S_3,c}}$	$\underline{A_{S_4,c}}$	$\overline{A_{S_4,c}}$
C_{11}	0.85	0.99	0.82	0.98	0.81	0.99	0.86	0.97
C_{12}	0.90	0.99	0.85	0.99	0.89	0.97	0.91	0.99
C_{13}	0.85	0.94	0.91	0.99	0.86	0.92	0.88	0.97
C_{21}	0.84	0.94	0.87	0.96	0.88	0.99	0.91	0.99
C_{22}	0.84	0.94	0.87	0.96	0.88	0.99	0.91	0.99
C_{23}	0.91	0.98	0.90	0.97	0.92	0.97	0.87	0.99
C_{24}	0.91	0.98	0.90	0.97	0.92	0.98	0.87	0.99
C_{31}	0.81	0.95	0.86	0.97	0.92	0.95	0.89	0.93
C_{32}	0.88	0.95	0.93	0.98	0.88	0.96	0.90	0.97

After the component weights from Table 10 are applied to Table 11 using Eq. (50), the Laplace criterion is used to determine the positive and negative ideal solutions found in Eqs. (70) and (71). These solutions are provided in order by criterion, or C_{11} through C_{32}. For example, for

component C_{23}, supplier S1 offers the part that provides the best and supplier S4 offers the least desired (weighted) interval availability.

$$\begin{aligned} B^{+} &= \{0.038, 0.161, 0.112, 0.007, 0.005, 0.019, 0.030, 0.803, 0.714\} \\ &= \{S_1, S_4, S_2, S_4, S_4, S_1, S_3, S_3, S_2\} \end{aligned} \tag{70}$$

$$\begin{aligned} B^{-} &= \{0.037, 0.156, 0.105, 0.007, 0.005, 0.019, 0.029, 0.756, 0.684\} \\ &= \{S_2, S_2, S_3, S_1, S_1, S_4, S_4, S_1, S_1\} \end{aligned} \tag{71}$$

To determine which supplier is ideal for *all* components, the separation between each alternative (supplier) and the B^+ and B^- suppliers is calculated using the Euclidean distance equations Eq. (53) and (54).

Table 12. Separation between each supplier and the ideal solutions.

Supplier	$\underline{D_b^+}$	$\overline{D_b^+}$	$\underline{D_b^-}$	$\overline{D_b^-}$
S_1	0.016	0.072	0.017	0.020
S_2	0.023	0.041	0.072	0.084
S_3	0.013	0.039	0.054	0.058
S_4	0.016	0.040	0.003	0.004

Finally, by applying Eq. (55), the supplier that is closest to the best availability for each component within the system is calculated. Supplier S_3 provides the best overall offerings of the nine components in the system according to the weighted importance of each component and would therefore be the best sole supplier of the components in the aircraft servo-actuation series-parallel system if availability is the primary metric of interest.

Table 13. Final supplier ranking.

Supplier	Laplace criterion $\left(\underline{D_b^\star} + \overline{D_b^\star}\right)$	Rank
S_1	0.631	4
S_2	1.596	3
S_3	1.857	1
S_4	1.607	2

3.5. Coherence of Interval Analysis and Comparison with Point Estimates

The supplier selection approach developed in the previous four sections is compared against a ranking of suppliers based on simulation, assuming known probability distributions rather than interval values. To do so, the component availability ranges described in Table 11 have been used to simulate the performance of the system in Figure 25 independently for each supplier and for 1000 simulation runs.

According to the ranking in Table 13, Supplier 3 is the preferred supplier, followed closely by Suppliers 4 and 2 and considerably ahead of Supplier 1. The distributions described in Figure 26, have been constructed in such a way by simulating (1000 times) component availability from each supplier following a uniform distribution as dictated by the bounds in Table 11. The results illustrate the distribution of availability for each supplier. The graphs clearly show that Supplier 1 provides the worst availability distribution. Supplier 3 is better that Supplier 4 since it has a smaller variance and the lower and upper ranges are higher. Finally, Supplier 2 is relatively as good but with a lower, lower bound and higher variance. Note that the difference in ranking for suppliers 2, 3 and 4 via Table 13 is quite small (around 10%), suggesting that the interval ranking approach allows for a coherent selection of supplier without resorting to numerous simulations. The same conclusions can be obtained when considering a triangular distribution with mid-point between the bounds presented in Table 11, shown in Figure 27.

Note that comparison is made here to the Laplace decision rule in Eq. (63), a somewhat risk neutral decision approach. Opting for the best or worst case rules could alter the decision throughout the steps, resulting in a different conclusion.

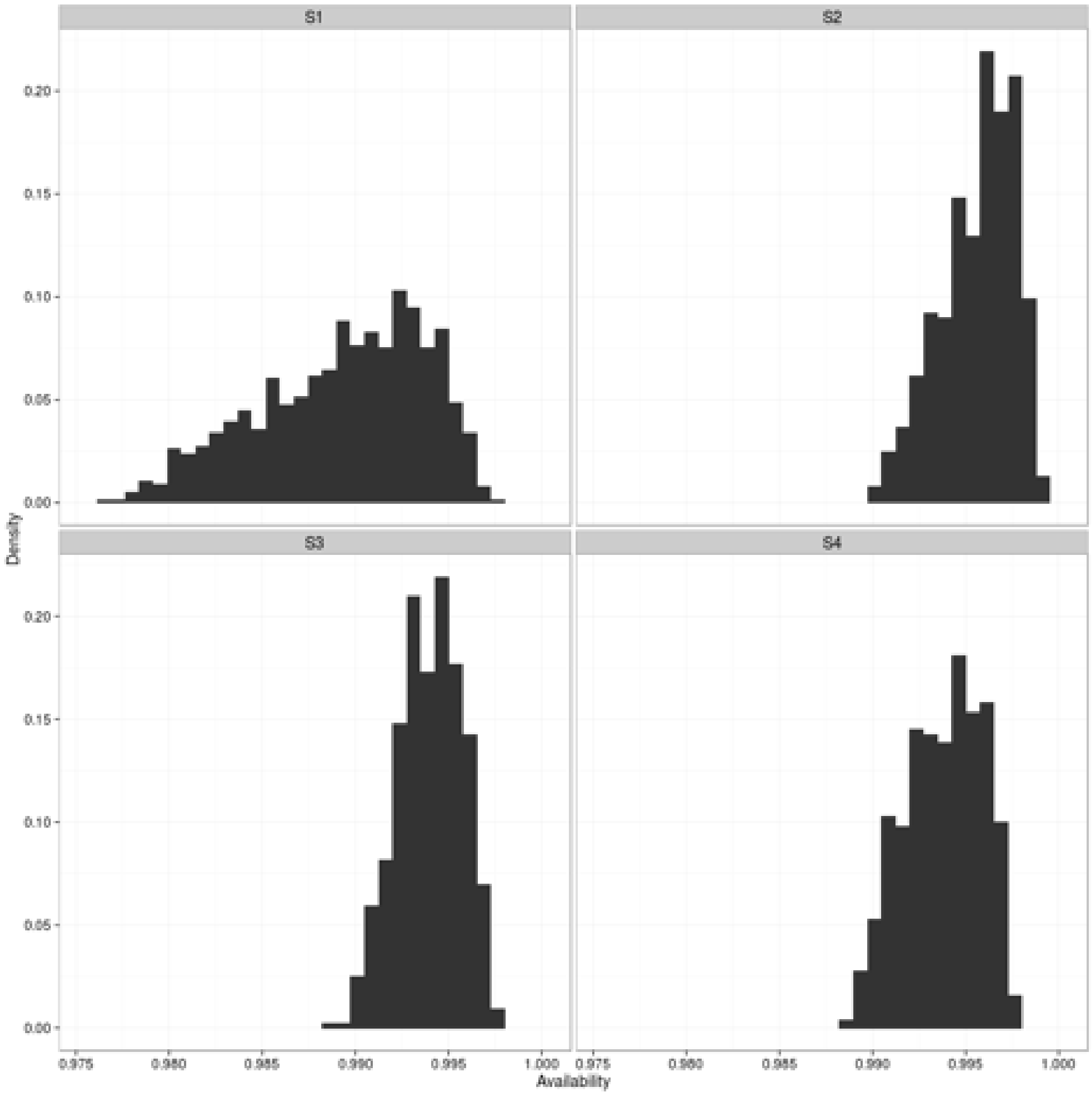

Figure 26. Availability comparison for the four suppliers, assuming a uniform distribution for availability uncertainty.

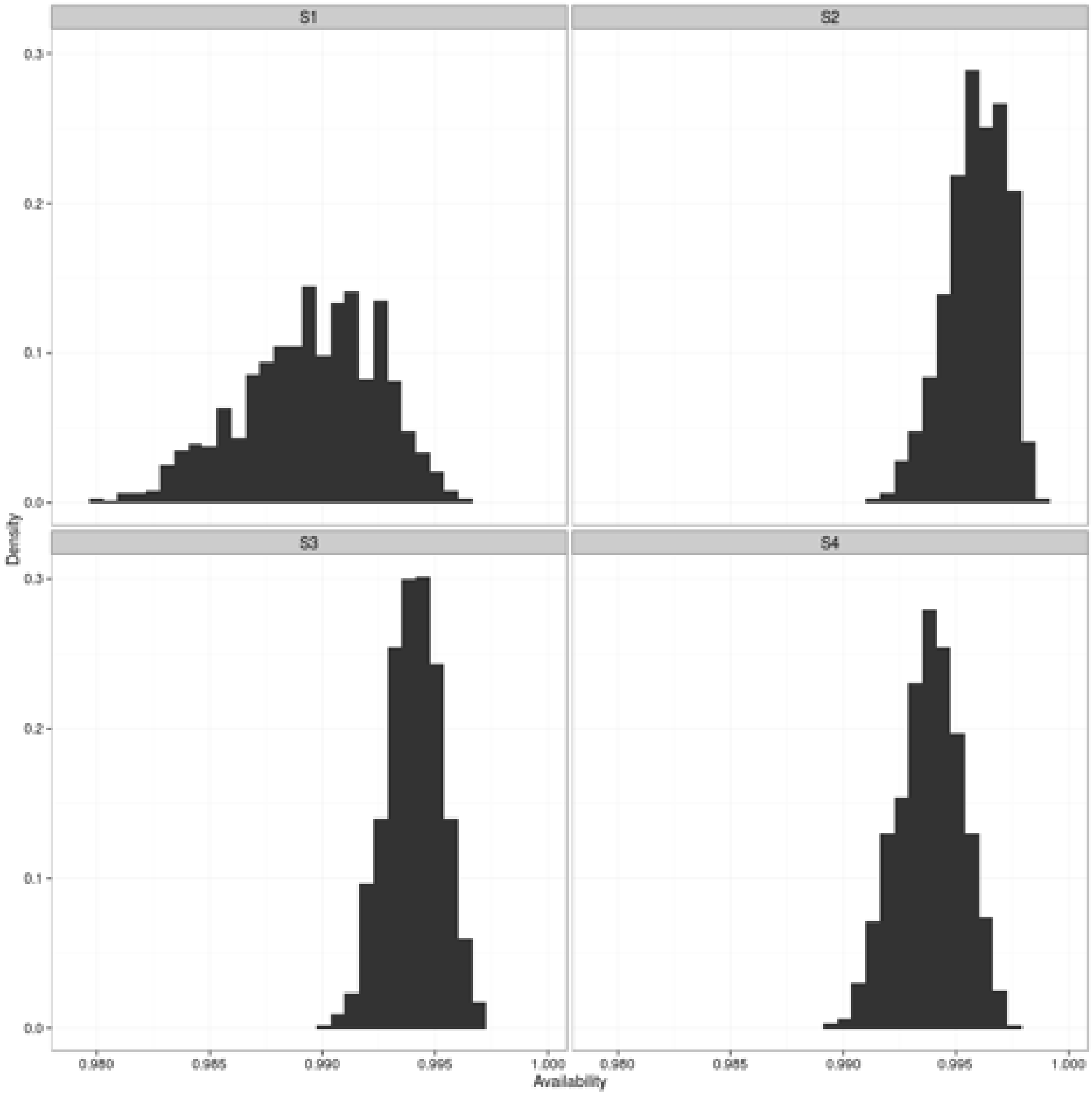

Figure 27. Availability comparison for the four suppliers, assuming a triangular distribution for availability uncertainty.

The Monte Carlo analysis assuming two common distributions used in expert elicitation resulted in the same supplier selection result as with the interval-valued analysis. Were point estimates used, assumed to be the mid-point of the intervals for time between failures and time to repair, the end supplier selection result is different. The component importance results for the interval calculation (from Table 9) and for the point estimate calculation are provided in Table 14. Very little difference is seen the importance ranking of the components, though there is a major difference in how the components are weighted based on their importance measure. The most preferred supplier does not change when the point estimate is used, though the preference order does change, as shown in Table 15. This suggests that limiting ourselves to the point estimate

can have a misleading impact on the choice of suppliers: the point estimate suggests that suppliers 1 and 4 are similar to each other when the interval result (and the simulation result) suggest something very different. This impact could be magnified given a potentially optimistic or pessimistic outlook (i.e., the best case or worst case decision rules in Eq. (63)).

Table 14. Comparison of interval and point estimate importance results.

	Interval uncertainty		Point estimate	
Component	Importance	Weight	Importance	Weight
C_{11}	0.0123	0.0208	0.000325	0.0310
C_{12}	0.0501	0.0850	0.002525	0.2410
C_{13}	0.0347	0.0589	0.001064	0.1015
C_{21}	0.0022	0.0037	1.19E-05	0.0011
C_{22}	0.0016	0.0028	6.41E-06	0.0006
C_{23}	0.0059	0.0101	1.15E-05	0.0011
C_{24}	0.0093	0.0157	1.46E-05	0.0014
C_{31}	0.2532	0.4295	0.003517	0.3356
C_{32}	0.2203	0.3736	0.003003	0.2866

Table 15. Comparison of interval and point estimate final supplier rankings.

	Interval uncertainty		Point estimate	
Supplier	$\left(\underline{D_b^\star} + \overline{D_b^\star}\right)$	Rank	$D_b^\star$	Rank
S_1	0.631	4	0.5741	3
S_2	1.596	3	0.2129	4
S_3	1.857	1	0.6391	1
S_4	1.607	2	0.5952	2

4. Concluding Remarks

There has been much research in the area of supplier selection, and much of which continue to follow Dickson's 23 criteria. The Department of Defense tends to consider procurement cost very strongly when considering suppliers of component parts for weapons systems, however another major source of subsequent costs is due to the unavailability of such systems. To be mission-ready, DoD systems must be available for use. As such, the objective of this paper is to provide an availability-based framework for choosing a supplier who can provide components that lead to a high system availability, focusing particularly on those components that are most important to system availability. Due to uncertainty in MTBF and MTTR component data, interval arithmetic provides a vehicle for making computations within a known range of data. Note that this is a fairly nuanced extension of a TOPSIS, though most any other multi-criteria decision analysis technique could take its place. Similarly, we focus on availability as a long-term cost driver for supplier selection, though other criteria could be considered in addition to or

in place of availability. A novel idea provided in this framework is the treatment of component performance as the criteria in the multi-criteria comparison, with weights being derived by component importance measures from the field of reliability engineering.

With the modern economy and the current budgetary constraints placed upon the DoD, obtaining components with a high availability and reliability is vital to efficiency. In the world of maintaining equipment, having fewer corrective repairs translates into more time for technicians to focus on other tasks such as preventive maintenance, which also is a proponent of equipment reliability. The effects of this element of "Better Buying Power" can be felt throughout the DoD in the form of reliability, availability, cost avoidance, and better resource allocation.

This work provides an important first step in integrating component importance into supplier selection. A primary limitation of this framework is the assumption that sole suppliers are chosen, though this could certainly be a realistic assumption at the subsystem level (as the type of system represented in Figure 25 would be one of many subsystems in a larger system). Future work will explore the development of a supplier mix to meet reliability and maintainability needs.

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